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Colloquium Speaker for 1932: Professor J. F. Ritt, Columbia University, New York City.

Chairman, Committee on Arrangements for the Semi-Centennial: Professor T. S. Fiske, Columbia University, New York City.

THE ANNUAL MEETING IN NEW ORLEANS

The thirty-eighth Annual Meeting of the American Mathematical Society was held at New Orleans, Louisiana, from Monday to Thursday, December 28–31, 1931. The meeting opened on Monday morning with a general session of the Society devoted to mathematical statistics, followed in the afternoon by sectional sessions. At the close of these, a meeting of the Council was held. There being no quorum at the meeting of the Board of Trustees announced for the same afternoon, the Board adjourned to meet in New York City on January 2.

On Tuesday morning, at a joint session with the American Physical Society, Professor G. D. Birkhoff spoke on *Stability and instability of physical systems*, and Dr. W. F. G. Swann on *The significance of the fundamental concepts of modern atomic theories*. The lecture by Professor Birkhoff will appear in an early issue of this Bulletin.

On Tuesday afternoon, at a joint session with the American Physical Society and Section A of the American Association for the Advancement of Science, Professor G. A. Bliss delivered his address as retiring vice-president of Section A, entitled *The calculus of variations of the quantum theory*. This was followed by the ninth Josiah Willard Gibbs Lecture, delivered by Professor P. W. Bridgman, and entitled *Statistical mechanics and the second law of thermodynamics*. Both of these lectures will appear in full in early issues of this Bulletin.

The annual business meeting of the Society, on Wednesday morning, preceded a general session devoted to papers on analysis. Wednesday afternoon was devoted to an excursion to the old French quarter of New Orleans. The annual dinner was held at the Patio Royale Restaurant on Wednesday evening, one hundred eighty being present and President Eisenhart acting as toastmaster; other speakers at the dinner were Professors H. E. Buchanan, Dunham Jackson, C. G. Latimer, Tibor Radó, and H. E. Slaught.

At the joint session of the Society and the Mathematical Association of America, on Thursday morning, Professor E. R. Hedrick delivered his retiring address as President of the

Society, entitled *Non-analytic functions of a complex variable*, and, at the request of the Mathematical Association, Professor R. D. Carmichael spoke on *Some researches in the theory of numbers*.

At the session on Wednesday morning, the Cole Prize in the Theory of Numbers was awarded to Professor H. S. Vandiver, of the University of Texas, for his several papers on Fermat's last theorem published in the Transactions of the American Mathematical Society and the Annals of Mathematics during the last five years, with special reference to a paper entitled *On Fermat's Last Theorem*, which appeared in volume 31 of the Transactions. Professor Vandiver then presented a résumé of this work.

The attendance included the following one hundred twenty-six members:

V. W. Adkisson, N. L. Anderson, B. M. Armstrong, C. S. Atchison, R. W. Babcock, E. A. Bailey, F. R. Bamforth, E. T. Bell, B. A. Bernstein, G. D. Birkhoff, G. A. Bliss, L. M. Blumenthal, H. E. Bray, H. E. Buchanan, W. D. Cairns, R. D. Carmichael, H. C. Carter, J. W. Cell, Jacques Chapelon, L. W. Cohen, J. B. Coleman, L. P. Copeland, A. T. Craig, D. R. Curtiss, H. T. Davis, C. H. Dix, B. F. Dostal, Otto Dunkel, W. L. Duren, Nat Edmonson, L. P. Eisenhart, G. C. Evans, G. W. Evans, B. F. Finkel, T. M. Focke, L. R. Ford, Tomlinson Fort, H. H. Germond, F. J. Gerst, Michael Goldberg, W. L. Hart, J. O. Hassler, E. R. Hedrick, D. M. Hickey, Einar Hille, Charles Hopkins, H. M. Hosford, J. C. Hughes, C. A. Hutchinson, Louis Ingold, M. H. Ingraham, Dunham Jackson, R. L. Jackson, B. W. Jones, F. W. Kokomoor, Alfred Korzybski, H. W. Kuhn, C. G. Latimer, F. A. Lewis, R. G. Lubben, Dorothy McCoy, E. J. McShane, L. A. MacColl, Israel Maizlish, A. J. Maria, T. E. Mason, E. D. Meacham, W. L. Miser, C. N. Moore, T. W. Moore, E. M. Morenus, Richard Morris, A. B. Morton, T. A. Mossman, E. J. Moulton, C. O. Oakley, J. O. Osborn, F. W. Owens, W. V. Parker, C. G. Phipps, Tibor Radó, S. E. Rasor, S. W. Reaves, D. P. Richardson, R. G. D. Richardson, H. L. Rietz, W. M. Rust, R. G. Sanger, I. J. Schoenberg, H. E. Schoonmaker, William Sell, F. R. Sharpe, C. R. Sherer, W. A. Shewhart, H. A. Simmons, T. M. Simpson, Edmond Siroky, H. E. Slaught, M. M. Slotnick, C. D. Smith, C. E. Smith, Herman L. Smith, Virgil Snyder, M. C. Spencer, R. C. Stephens, R. W. Stokes, W. F. G. Swann, J. L. Synge, J. D. Tamarkin, J. M. Thomas, J. F. Thomson, H. W. Tyler, H. S. Vandiver, R. W. Veatch, Oswald Veblen, M. E. G. Waddell, J. H. Weaver, W. P. Webber, J. H. M. Wedderburn, F. M. Weida, M. J. Weiss, K. P. Williams, H. A. Wilson, W. H. Wilson, Kathryn Wyant, Zarmair Zakarian.

The following were elected to Sustaining Membership:

The University of Chicago;
Members of the Department of Mathematics, George Washington University.

The following persons were elected to membership in the Society:

Mr. John Landes Barnes, Princeton University;
Professor James Holley Bartlett, Jr., University of Illinois;
Mr. James Francis Butler, St. Ignatius High School, Chicago;
Mr. Joseph Canning, Helena, Montana;
Professor Fitzhugh L. Carmichael, Denver School of Commerce;
Professor John Wesley Cell, Southern Methodist University;
Mr. Mannis Charosh, New Utrecht High School, Brooklyn;
Professor Lena Rachel Cole, Central Normal College, Danville, Indiana;
Mr. José Isaac Corral-Alemán, Director de Montes y Minas de la República de Cuba;
Mr. Rustom D. Dalal, Bombay, India;
Mr. Jared William Davis, Dorchester High School for Boys, Dorchester, Massachusetts;
Mr. Francis George Dressel, Duke University;
Mr. Edward Mark Evarts, patent attorney, New York City;
Mr. Howard Gray Funkhouser, Columbia University;
Mr. Arthur Haas, Thomas Jefferson High School, Brooklyn;
Mr. Richard Charles Hildner, Ohio State University;
Dean Clarence Francis Holmes, Texas College;
Lieutenant Colonel Harris Jones, United States Army, professor of mathematics, United States Military Academy, West Point;
Mr. Howard Scholl Kaltenborn, Carnegie Institute of Technology;
Dr. Egbert R. van Kampen, Johns Hopkins University;
Professor A. Collins Ladner, Denison University;
Mr. Walter S. Lawton, Philadelphia;
Mr. Franklin Stuart Lerch, Union College;
Mr. Daniel Boone Lloyd, Syracuse University;
Dr. Donald L. McDonough, Drexel Institute;
Dr. William Markowitz, Pennsylvania State College;
Mr. Francis Philip Nash, Groton School;
Miss Marie Margaret Ness, University of Minnesota;
Dean Olaf Morgan Norlie, Hartwick College;
Mr. Jonathan Pickett, Realitos, Texas;
Professor Herbert ReBarker, East Carolina Teachers College;
Miss Mary Sophia Sabin, Point Loma, California;
Dean Richard E. Scammon, University of Minnesota;
Mr. Robert Elliott Street, Rensselaer Polytechnic Institute;
Professor Althéod Tremblay, Laval University;
Miss Alice Willard Turner, Wheaton College;
Mr. Emil Fridstein Vaage, American Telephone and Telegraph Company, New York City;
Mr. Henry Walther, Bell Telephone Laboratories, New York City;
Professor Charles H. Wheeler, University of Richmond;

As Nominees of Members of the Department of Mathematics of the University of Wisconsin: Mr. Ray Moltzner Ely, Mr. Albert E. May, Mr. Eugene A. Rasor, Reverend Robert Albert Sromovsky, Miss Louise A. Wolf;

As Nominees of the University of Michigan: Mr. John R. Abernethy, Mr. Clevoe D. Jones, Mr. Henry Van Engen.

The ordinary membership in the Society is now 1883, including 84 nominees of sustaining members and 83 life members. There are also 22 sustaining members. The total attendance of members at all meetings in 1931 was 837; the number of papers read was 374; the number of members attending at least one meeting, 578.

At the annual election, which closed on December 30, and at which 230 votes were cast, the following officers and members of the Council were elected:

Vice-Presidents, Professors E. W. Chittenden and H. H. Mitchell.

Secretary, Dean R. G. D. Richardson.

Treasurer, Professor G. W. Mullins.

Associate Secretaries, Professors M. H. Ingraham and T. M. Putnam.

Member of the Editorial Committee of the Bulletin, Professor W. R. Longley.

Member of the Editorial Committee of the Transactions, Professor J. D. Tamarkin.

Member of the Editorial Board of the American Journal of Mathematics, Professor A. B. Coble.

Member of the Editorial Committee of the Colloquium Publications, Professor R. L. Moore.

Members of the Council, Professors C. R. Adams, T. R. Hollcroft, C. C. MacDuffee, W. E. Milne, and E. H. Moore, to serve three years; Professor Arnold Emch, to serve two years.

The reports of the Treasurer and of the auditors (Professor H. W. Reddick and Mr. J. J. Tanzola) showed a balance of $5303.02, exclusive of the balances in the Bulletin, Transactions, Colloquium, Journal, Library, and special funds. The Society's Endowment Fund, invested in securities of par value $76,000, yielded in 1931 a net income of $3529.53; sustaining memberships for the year amounted to $2715. The amount received

from sales of the Society's publications was $9994.29. During the year special contributions from members were received to the amount of $325. The trustees adopted a budget for 1932 showing estimated expenditures and receipts as $40,569.13 and $38,174.13 respectively.

The President and Secretary were requested to solicit special contributions to help balance the budget for 1932.

The Librarian reported that the Library of the Society now has 7480 books. A new catalogue has been prepared, and has already appeared as supplement to the January number of this Bulletin.

The following appointments were reported: as tellers for the election at the annual meeting, Professors A. B. Morton and P. A. Smith; as an additional member of the Committee on the Semi-Centennial, Professor G. W. Mullins; as representative on the National Research Council for the period beginning July 1, 1932, and to succeed Professor James Pierpont, Professor A. B. Coble; as the committee on nomination of the Gibbs Lecturer for 1933, Professors Oswald Veblen (chairman), A. B. Coble, J. L. Coolidge, F. D. Murnaghan, and K. P. Williams; as committee on arrangements for the annual meeting of 1932, Professors J. R. Kline (chairman), M. S. Knebelman, Richard Morris, R. G. D. Richardson, and H. P. Robertson; as representatives on the Council of the American Association for the Advancement of Science, Professors Louis Ingold and M. H. Ingraham; as the nominating committee, Professors R. L. Moore (chairman), T. H. Hildebrandt, Solomon Lefschetz, W. A. Manning, and Virgil Snyder.

The Society passed votes of thanks to Professor Nola Lee Anderson, Professor H. E. Buchanan, Dean Pierce Butler, President Albert B. Dinwiddie, Professor William Duren, Professor Anna Many, Professor R. L. Menuet, and Professor Marie Weiss for their efficient work in making local arrangements for the meeting of the Society.

A resolution of appreciation of the work of Professor J. W. Young as chairman of the committee on funds, and of sympathy in his illness, was passed by the Council and also by the mathematicians at the joint dinner.

The annual meeting of the Society for 1932 was fixed for De-

cember 28–30, 1932, at Atlantic City and Princeton University. The tenth Josiah Willard Gibbs Lecture will be delivered at Atlantic City.

Titles and cross-references to the abstracts of papers read at the regular sessions follow below. Mr. Fischer was introduced by Professor Rietz, and Dr. Hopf by Professor Tamarkin. The papers whose abstract numbers are followed by the letter *t* were read by title.

General Session, Statistics, Monday Morning.

1. *On the Lexis theory and the analysis of variance*, by Professor H. L. Rietz. (Abstract No. 38–1–7.)

2. *A modification of Stirling's formula arising from an asymptotic evaluation of the Wallis integral*, by Professor H. H. Germond. (Abstract No. 38–1–4.)

3. *Another coefficient of linear correlation between two variables*, by Professor J. B. Coleman. (Abstract No. 38–1–5.)

4. *On certain non-normal correlation surfaces*, by Mr. C. H. Fischer. (Abstract No. 38–1–6.)

5. *The simultaneous distribution of mean and standard deviation in small samples*, by Dr. A. T. Craig. (Abstract No. 38–1–8.)

6. *A closed expression approximating to the value of the probability integral*, by Professor H. H. Germond. (Abstract No. 38–1–9–*t*.)

7. *On the correlation between certain averages*, by Dr. A. T. Craig. (Abstract No. 38–1–10–*t*.)

8. *Frequency laws showing stability with reference to the geometric mean and other means*, by Professor E. L. Dodd. (Abstract No. 38–1–11–*t*.)

9. *Distributions of certain statistics in matched pairs of samples from a normal bi-variate population*, by Dr. S. S. Wilks (National Research Fellow). (Abstract No. 38–1–12–*t*.)

10. *Determination of characteristic functions of quadratic statistics by determinants*, by Dr. S. S. Wilks (National Research Fellow). (Abstract No. 38–1–13–*t*.)

11. *A generalized error function*, by Mr. Albert Wertheimer. (Abstract No. 38–1–14–*t*.)

Section I, Algebra and Geometry, Monday Afternoon.

12. *On the class number of an abelian field*, by Professor C. G. Latimer. (Abstract No. 38–1–15.)

13. *On the reduction of positive quaternary quadratic forms* (preliminary communication), by Professor B. W. Jones. (Abstract No. 38–1–16.)

14. *Two involutorial transformations, of orders* 11 *and* 9, *associated with null reciprocities*, by Professors Hazel E. Schoonmaker and Virgil Snyder. (Abstract No. 37–11–404.)

15. *The quartic space involutorial transformations with a double conic*, by Dr. L. A. Dye and Professor F. R. Sharpe. (Abstract No. 38–1–17.)

16. *On fundamental transformations of surfaces*, by Dr. M. M. Slotnick. (Abstract No. 38–1–18.)

17. *Isometric properties of projective transformations* (preliminary communication), by Professor L. R. Ford. (Abstract No. 38–1–19.)

18. *The isoperimetric problem for polyhedra*, by Mr. Michael Goldberg. (Abstract No. 37–11–403.)

19. *On the isodynamic points of four spheres*, by Professor Nathan Altshiller-Court. (Abstract No. 38–1–20–*t*.)

20. *Semi-linear equations*. Part II: *Irregular polygons*, by Professor C. O. Oakley. (Abstract No. 38–1–21.)

21. *Generalizations of the author's earlier results relative to the diophantine equation* $\sum(1/(x_1x_2 \cdots x_r)) = b/a$, by Professor H. A. Simmons. (Abstract No. 38–1–55.)

22. *Apolarity in the Galois fields of order* 2^n, by Professor A. D. Campbell. (Abstract No. 38–1–23–*t*.)

23. *Normal division algebras of degree* 2^e *over an algebraic field*, by Professor A. A. Albert. (Abstract No. 38–1–24–*t*.)

24. *Partitionment of representations in indefinite binary quadratic forms*, by Dr. Gordon Pall. (Abstract No. 38–1–25–*t*.)

25. *A class of universal functions*, by Dr. Gordon Pall. (Abstract No. 38–1–26–*t*.)

26. *Properties of highly composite numbers*, by Professor R. G. Archibald. (Abstract No. 38–1–27–*t*.)

27. *A type of conformality in three dimensions*, by Professor I. M. Sheffer. (Abstract No. 38–1–28–*t*.)

28. *A new set of independent postulates for the algebra of logic*, by Professor E. V. Huntington. (Abstract No. 38–1–54–*t*.)

Section II, Applications, Point Sets, and Foundations, Monday Afternoon.

29. *The apsides of general dynamical systems*, by Professor J. L. Synge. (Abstract No. 38–1–29.)

30. *A discontinuous dynamics*, by Dr. C. H. Dix. (Abstract No. 38–1–30.)

31. *On the time-average theorems in dynamics*, by Dr. Eberhard Hopf. (Abstract No. 38–1–31.)

32. *A complete characterization of proper pseudo d-cyclic sets of points*, by Dr. L. M. Blumenthal. (Abstract No. 38–1–32.)

33. *Continuous transformations of abstract spaces*, by Dr. Rothwell Stephens. (Abstract No. 38–1–33.)

34. *A non-aristotelian system and its necessity for rigor in mathematics and physics*, by Mr. Alfred Korzybski. (Abstract No. 38–1–34.)

35. *Thermostatic control*, by Professor J. L. Synge. (Abstract No. 38–1–35–*t*.)

36. *Generalizations of time in dynamics*, by Dr. C. H. Dix. (Abstract No. 38–1–36–*t*.)

37. *Some additions to the theory of combinators*, by Dr. H. B. Curry (National Research Fellow). (Abstract No. 38–1–37–*t*.)

38. *On the decomposability of closed sets into a countable number of simple sets of various types*, by Dr. G. T. Whyburn. (Abstract No. 37–11–405–*t*.)

39. *On sets of constant dimension*, by Professor R. G. Putnam. (Abstract No. 38–1–38–*t*.)

40. *Conditions that a graph have a dual*, by Dr. Hassler Whitney (National Research Fellow). (Abstract No. 38–1–39–*t*.)

41. *Basic graphs*, by Dr. Hassler Whitney (National Research Fellow). (Abstract No. 38–1–40–*t*.)

42. *A set of topological invariants for graphs*, by Dr. Hassler Whitney (National Research Fellow). (Abstract No. 38–1–41–*t*.)

43. *Characteristic functions and the algebra of logic*, by Dr. Hassler Whitney (National Research Fellow). (Abstract No. 38–1–42–*t*.)

44. *The problem of the sheet*, by Dr. D. G. Bourgin. (Abstract No. 38–1–53–*t*.)

General Session, Analysis, Wednesday Morning.

45. *Divergent series and singular points of ordinary differential equations*, by Professor F. R. Bamforth. (Abstract No. 38–1–43.)

46. *On the semi-continuity of double integrals in the calculus*

of variations, by Dr. E. J. McShane (National Research Fellow). (Abstract No. 37–7–254.)

47. *A functional equation involving well known special cases*, by Professor W. H. Wilson. (Abstract No. 38–1–44.)

48. *A singular case of the Laplace differential equation of infinite order*, by Professor H. T. Davis. (Abstract No. 38–1–45.)

49. *Pfaffian systems of species one*, by Professor J. M. Thomas. (Abstract No. 38–1–46.)

50. *A generalization of Weierstrass's and Fekete's mean-value theorems*, by Professor Morris Marden. (Abstract No. 38–1–47–*t*.)

51. *Concerning a certain class of linear inequalities connected with the theory of completely monotonic functions*, by Dr. I. J. Schoenberg. (Abstract No. 38–1–48.)

52. *A type of non-absolutely convergent integral*, by Professor L. W. Cohen. (Abstract No. 38–1–49.)

53. *Note on the stereographic definition of plurisegments on the hypersphere*, by Professor Nat Edmonson. (Abstract No. 38–1–50.)

54. *On Riesz and Cesáro methods of summability*, by Dr. R. P. Agnew (National Research Fellow). (Abstract No. 38–1–51–*t*.)

55. *Note on Perron's solution of the Dirichlet problem*, by Dr. Hassler Whitney (National Research Fellow). (Abstract No. 38–1–52–*t*.)

TOMLINSON FORT,
Associate Secretary

THE SCIENTIFIC WORK OF C. L. E. MOORE

In the death of Clarence Lemuel Elisha Moore the Massachusetts Institute of Technology loses an outstanding educator, and the United States one of its most prominent geometers. Born in Bainbridge, Ohio, May 12, 1876, from settler stock, he decided upon a career of study, and received the degree of B.Sc. from the Ohio State University in 1901, the degree of A. M. from Cornell in 1902, and the degree of Ph.D. from Cornell in 1904. He also studied at the universities of Göttingen, Turin, and Bonn, where he came under the influence of Segre and Study. In 1904 he joined the staff of the Massachusetts Institute of Technology, where he taught successively as instructor, assistant professor, associate professor, and full professor. He died December 5, 1931, after a surgical operation. He is survived by his wife, Belle Pease Fuller, and a daughter, Hazel.

As a member of the teaching staff at the Massachusetts Institute of Technology he was responsible for a great deal of the increased mathematical activity at this institute in the last twenty years, not only through his own research but also in his constant encouragement of the work of others. Students and members of the staff liked to talk things over with Moore, and many a scientific or educational opinion has ripened in the discussion with him.

His mathematical work shows a remarkable unity, combined with intuition for living problems and elegance of geometric conception. Starting from problems on algebraic geometry in ordinary space, following Klein, Segre, and Study, he soon became interested in spaces of more dimensions, then in the differential geometry of such spaces, and at last in the differential geometry of Riemannian manifolds. He belongs, with Ricci, for whom he had a great admiration, to the group of geometers who had the satisfaction of seeing the usefulness of their efforts recognized by later developments. With Ricci and Study he also shared the Leibnizian conviction of the importance of the formal side of the computation, and to this we owe work (in collaboration with H. B. Phillips) in vector analysis and related subjects.

Some of his major contributions to geometry are the investigation of minimal planes, his theory (together with Phillips) of linear distance and angle in projective geometry, his theory (together with E. B. Wilson) of two-dimensional surfaces in hyperspace, his study of remarkable surfaces in hyperspace (rotation surfaces, minimal surfaces, developable surfaces) and of interesting curves on manifolds lying in hyperspace. A series of investigations (together with Philip Franklin) on the geometric properties of Pfaffians occupied his mind at the time he was stricken.*

Though deeply interested in those modern investigations which tend to broaden the concept of differential geometry by the study of different dis-

* A detailed analysis of Moore's personality and scientific work will appear in the Journal of Mathematics and Physics, of which he was one of the founders.

placements, Moore devoted most of his research to the study of the many unknown properties of euclidean and riemannian hyperspace. The deep scientific wisdom of this decision becomes clear only to him who realizes how little we know of the detailed aspects of such manifolds, how surprising the results may be, and how important a thorough investigation is, not only for differential geometry, but also for topology, calculus of variations, and even physics. In this field Moore stands as sturdy a pioneer as his ancestors were in the woods of the Middle West.

D. J. STRUIK

ENCICLOPEDIA ELEMENTARI

Enciclopedia delle Matematiche Elementari a cura di L. Berzolari, G. Vivanti e D. Gigli. Milan, Hoepli. Volume 1, Part 1, 1930, xvi+450 pp.; Volume 1, Part 2, 1932, xvi+609 pp.

The preparation of an Italian encyclopedia of elementary mathematics was approved by the Society Mathesis as early as 1909, and the project was then viewed with favor also in other countries so that it seemed desirable to publish a German edition contemporaneously with the original. Active preparation was begun at once and in 1916 the proposed publication was announced in my *Historical Introduction to Mathematical Literature*, page 279, as well as in other places. Various obstacles, including the World War, delayed the publication so that the first volume, which is devoted to analysis, was only recently completed after being brought up to date by the authors of the articles contained therein. The remaining two volumes are to treat geometry and applied mathematics respectively. The latter volume is expected to include also a discussion of the teaching and the history of mathematics.

The twenty main headings of the present volume, with the number of pages devoted to each of these subjects and the authors of the articles, are as follows: *Logic*, (75), A. Padoa; *General arithmetic*, (126), D. Gigli; *Practical arithmetic*, (52), E. Bortolotti and D. Gigli; *Theory of numbers and indeterminate analysis*, (68), M. Cipolla; *Progressions*, (17), A. Finzi; *Logarithms*, (42), A. Finzi; *Mechanical calculus*, (28), G. Tacchella; *Combinatory calculus*, (9), L. Berzolari; *Elements of the theory of groups*, (51), L. Berzolari; *Determinants*, (30), L. Berzolari; *Linear equations*, (13), L. Berzolari; *Linear substitutions and linear, bilinear and quadratic forms*, (28), L. Berzolari; *Rational functions of one or more variables*, (37), O. Nicoletti; *General properties of algebraic equations*, (59), O. Nicoletti; *Equations of the second, third, and fourth degree, and other particular algebraic equations, systems of algebraic equations of elementary type*, (57), E. G. Togliatti; *Methods for the discussion of problems of the second degree and remarks on some of the third and fourth degree*, (63), R. Marcolongo; *Limits, series, continued fractions, and infinite products*, (45), G. Vitali; *Elements of infinitesimal analysis*, (101), G. Vivanti; *Relations between the theory of aggregates and elementary mathematics*, (11), G. Vivanti; *The analytic function from an elementary point of view*, (29), S. Pincherle.

Each of the two parts of this volume closes with an author index, but the volume contains no subject index. It contains an unusually large number of references to sources and hence it is very useful to those interested in the history of elementary mathematics. The large amount of space devoted to such subjects as infinitesimal analysis and the theory of groups exhibits the fact that modern developments receive considerable attention and that the authors do not share the view that the developments of elementary mathematics were practically completed at the close of the seventeenth century. On the contrary, they point out the well known fact that many of the developments in modern advanced mathematics extend into the elements of our subject and throw new

and useful light on these elements. Fifty years ago not many mathematicians would have been so bold as to predict that in 1931 an encyclopedia of elementary mathematics would devote more than fifty pages to algebraic group theory, as is done in the present work.

This work differs widely from the only other large modern encyclopedia of elementary mathematics, by Weber and Wellstein. It is much less in the form of a treatise on the subjects concerned, but it aims to prove the most fundamental theorems and to give abundant references to places where the proofs of others can be found, especially the earliest ones. By limiting itself to the elements, the exposition becomes comparatively brief, and this enables the reader to gather here with comparative ease the main facts relating to a wide range of subjects. The subject which is treated at the greatest length is general arithmetic, and the subject to which the least number of pages is devoted is combinatory calculus. The work is especially useful to the younger students of our subject who desire to become acquainted with the main elements of the various fields of mathematics before entering deeply into any one of them. It should also be very useful to teachers of secondary mathematics who can thereby find a large amount of modern material and of historical data relating to the subjects which they are teaching.

Near the beginning of each of the two parts of the present volume there appears a list of mathematical publications with the abbreviations therefor used in the body of this work. The second part contains 155 such publications but only 10 of these are American. The most conspicuous of the missing ones is the Proceedings of the National Academy of Sciences, which now contains a large number of mathematical articles. On page 410 and in the index of Part 1 there appears the name F. R. Moulton instead of J. F. Moulton, and on page 19 of Part 2, the date of Ruffini should be 1799 in place of 1798. On page 43 of this part it is stated that the natural numbers form an infinite abelian group when they are combined by multiplication. This error was, however, corrected, even before it was made, on page 33, note 77. The same error appears also in the first edition of volume 2 of Weber's *Lehrbuch der Algebra*, as well as in many other places. On page 314 and in the index the name J. Kempner should be replaced by A. J. Kempner. The number of such oversights is fortunately relatively small, so that the volume as a whole can be heartily recommended. It would be very desirable to have a similar work in English notwithstanding the fact that a considerable portion of the modern mathematical language is universal and hence English speaking students of mathematics can use this work without much language trouble.

G. A. MILLER

LORIA ON HISTORY OF GEOMETRY

Il Passato e il Presente delle principali Teorie Geometriche. Storia e Bibliografia. Quarta edizione totalmente rifatta. By Gino Loria. Padua, Casa Milani ("Cedam"), 1931. xxiv+467 pp. 60 lire.

This work has been so long and favorably known, having already passed through three editions, that in the brief space allowed under present conditions for a review it will serve our purposes if attention is called to only a few of its leading features. It is divided into two books, (1) From the beginning of geometric research to the close of the 19th century; and (2) The progress of geometry in the last thirty years. Of these, the second will prove the more interesting to most readers because the field is new and the sources of information, classified in subsections, are not so generally known as those of the period before 1901. A brief statement of the chapters in the first book will, however, serve two useful purposes; it will show to those not already familiar with the earlier editions the general nature of the work as a whole, and at the same time it reveals the foundation upon which the later structure rests. Condensing the chapter titles, we may say that the first book covers the following general topics: (1) General view of the development of geometry up to about 1850, (2) Theory of algebraic plane curves, (3) Theory of algebraic surfaces, (4) Theory of algebraic curves of double curvature, (5) Differential geometry, (6) Forms of geometric figures, curves of higher orders, analysis situs, (7) Modern geometry of space, congruences, (8) Correspondences, projections, transformations, (9) Numerative geometry, (10) Non-euclidean geometry, (11) Geometry in n-dimensional space, (12) General summary. This part of the work is almost identical with the whole of the second edition (1896) and the major portion of the third (1907).

The second book consists of three chapters with various subdivisions comprising substantially the same topics as in chapters 1–12.

What will first strike all readers is the great amount of periodical literature examined and classified. That this is confined almost entirely to Italian, English, German, and French publications is to be expected, since it is here that the largest number of original articles have appeared, and since these are the only languages familiar to most scholars. There are also references, however, to a few periodicals in the Scandinavian languages, the Dutch, and the Japanese (articles in English, French, or German). Professor Loria has examined upwards of a hundred journals and has classified the articles relating to geometry in such a way that students can find sources with a minimum of difficulty.

In the second book he begins by paying tribute to the achievements of those great leaders in the domain of geometry who passed away in this thirty-year period, with a few names belonging near the close of the preceding century. In Italy there was that "noble triad" consisting of Brioschi, Beltrami, and Cremona; there was "Cesàro, nel fiore di una meravigliosa attività scientifica"; and still later there followed Veronese, Dini, Segre, and Bianchi. Germany lost

such men as Kronecker and Weierstrass; later, Reye and Sturm, "degni eredi della tradizione geometrica steineriana"; and finally Schwarz and Klein, "dei quali è superfluo ricordare le alte benemerenze scientifiche." England had hardly recovered from the death of Cayley when she was called upon to mourn the loss of that "glorious triad" of Sylvester, Salmon, and Kelvin. France, to take but one other country, was called upon in this period to witness the departure of such great analysts as Bertrand, Hermite, and Jordan; and later of Poincaré and Darboux who contributed so strikingly to all branches of mathematics.

Professor Loria then surveys rapidly the significant contributions to one or two typical geometric problems,—the Poncelet polygons and, more generally, to the problems of closure, to the imaginary in geometry, and to various fundamental concepts, all made in the border years between the last century and this. In both parts of the work the purpose has been to give to the reader a rather exhaustive list of references instead of furnishing him with digests of the contents of the articles mentioned. For this list the student will be thankful.

One feature of the work is of particular interest to American scholars. Altogether there are somewhat more than a hundred names of members of the American Mathematical Society in the lists of authors of monographs to which reference is made. Substantially all of these articles are mentioned only in Book II; that is, these contributors to the single field of higher geometry outnumber all those of equal productive ability in this country, in the entire field of advanced mathematics, prior to the twentieth century.

In the earlier part of the work no serious attempt has been made to list the latest discoveries with respect to Babylonian, Sumerian, and Egyptian mathematics.

The most disagreeable duty of any reviewer is to call attention to errata, unless these concern matters of fact. Professor Loria mentions only three of this type, and no large number would be apt to enter into a book that is chiefly bibliographical. The misprints, however, are very numerous, and for purposes of reference the book is made less valuable thereby. It suffices to mention only a few of these errors to show the necessity for care in quoting authorities. For example, we have such proper names as Mange, Ronse Ball, Zeuthon, Sanguli, G. A. Scott for C. A. Scott, Snider for Snyder, and Kanser for Kasner; such mistakes in French accents as are seen in dècouverte, problémes, lumiére, and thêorie and theorie; and such slips in spelling as téhorème, moyns for moyen, systhem, mahtematics, Edinburg, spharischen, entwichelungen, Dreiek, Steinerion, Malh. for Math., and Kurcen for Kurven. Besides all these there are many errors in dates, such as (Fermat) 1655 for 1665; (van Schooten) 1661 for May 29, 1660; (Halley) 1724 for 1742; (Clairaut) 1715 for 1713; (D'Alembert) 1716 for 1717; (Laplace) 1743 for 1749, and others like 1808 for 1908, and 1821 for 1921.

I have occasionally called attention to the unusually large number of misprints in French and Italian works, apparently due to the fact that proofreading is looked upon as the printer's concern rather than the author's. It is unfortunate.

DAVID EUGENE SMITH

BRUNET ON THE NEWTONIAN MOVEMENT

L'Introduction des Théories de Newton en France au XVIII*e Siècle. Avant* 1738. By Pierre Brunet. Paris, Blanchard, 1931. vii+355 pp. 55 francs.

The author of this work, already well known for his treatise on the Dutch physicists of the 18th century (1926) and his biography of Maupertuis (1929), has here entered upon a new and important field of investigation. In it he carries his studies of Newton's theories (chiefly gravitation) up to the date of the publication of Voltaire's *Éléments de la Philosophie de Newton* (1738, 2 edns.), planning to cover the period from 1738 to the end of the century in a second volume now in process of completion. He thus brings the study, for the moment, up to the period when Newton began to become more generally known in France owing largely to the influence of Maupertuis and Voltaire, an influence which later led to the work of du Châtelet (posthumously published in 1759) and to the appreciation of the discoveries of the British savant by the remarkable coterie of French mathematicians which appeared in the half century following. The study is especially valuable in that it shows the nature, the source, and the intensity of the French opposition to the theories of Newton in the period under discussion. It is divided into five chapters, the nature of which may briefly be stated as follows:

Chapter I considers the Cartesian opposition (1700–1720), beginning with the *Nouveau Système ou Nouvelle Explication du Mouvement des Planètes*, by Abbé Villemot (Lyon, 1707), which was an outgrowth of the well known theory of vortices, and which led to some interesting correspondence between Leibniz and Jean Bernoulli. After referring to the discussions of Fontenelle and Cassini I, M. Brunet proceeds to review the Cartesian theory as considered by Saurin (1702–1709), Fontenelle (1708, following his earlier contributions to the history of the Académie des Sciences), Cassini I (1710 and later), Saulmon (1709), and the Italian scientist Poleni in his *Dialogus de Vorticibus Coelestibus*, which was published at Padua in 1712. No one of these men had the scientific ability to handle the problem, and the arguments of Poleni and others were immediately confuted by Cotes in the preface to the second edition of the *Principia* (1713).

Chapter II discusses "Les préliminaires du débat,"—the years 1720–1728 when the *Optics* of Newton and the works of Gregory (*Astronomiae Physicae et Geometriae Elementa*, Oxford, 1702) and Keill (*Introductio ad Veram Physicam*, Oxford, 1700) were beginning to be appreciated across the Channel. The latter work was edited by s'Gravesande in 1725, and Newton's *Optics* was translated by a new and powerful protagonist, Coste, whose edition appeared in 1720. The battle was now beginning.

Chapter III, "L'effort des grands cartésiens (1728–1732)," describes what would seem to a modern scientist reminiscent of the Eastern battle of the pigmies and the giants; but the Cartesian cause, in so far as it was anti-newtonian, was already lost, although the enemy did not realize the fact.

Chapter IV, "Les premiers travaux newtoniens (1732–1734)," describes the real beginning of the Newtonian advance. A new comet had appeared above the horizon in the person of the young Pierre-Louis Moreau de Maupertuis, a man who knew more mathematics than any of the anti-newtonians, perhaps excepting Jean Bernoulli, and who had the daring and the vigor of youth. The story of his early championship of Newton's theories is told by Fontenelle in his *Histoire de l'Académie des Sciences* (1732, p. 132), and it was this championship that placed him at the head of the Lapland expedition as one of the "earth flatteners." It was not without a recognition of the danger of his position that he took a stand that "me fit des ennemis personnels dans ma patrie." D'Alembert later recalled these words and wrote: "Le premier qui ait osé parmi nous se déclarer ouvertement newtonien est l'auteur du *Discours sur la Figure des Astres*, qui joint à des connaissances géométriques très étendues cet esprit philosophique avec lequel elles ne se trouvent pas toujours et ce talent d'écrire auquel on ne croira plus qu'elles nuisent quand on aura lu ses ouvrages." It is here, then, that the first decided step was taken to make Newton's philosophy known in a country that has usually shown some hesitancy in accepting the views of her western neighbors. The second step was taken by the greatest of the early French admirers of Newton, Voltaire, whose *Lettres Anglaises* (1732, published in 1733) first called the attention of the intelligentsia of Paris to the nature of the former's works.

Chapter V, "La préparation des grandes controverses (1735–1737)," closes the epoch which preceded the publication of Voltaire's work on the subject and the beginning of the labors of du Châtelet and others who gave to France at first hand, and in the vernacular, the work of the great English genius. It was more than twenty years before the publication of du Châtelet's work that Voltaire saw "a professor of mathematics, only because he was great in his vocation," buried "like a king who had done good to his subjects."

The author has supplied a good bibliography of French and Latin works upon the subject omitting any reference to works in English (and perhaps excusably), and gives a helpful "Table des matières;" but, as usual in such publications from the Paris presses, there is no index,—a feature which is always a great desideratum in works of reference.

DAVID EUGENE SMITH

COOLIDGE ON ALGEBRAIC CURVES

A Treatise on Algebraic Plane Curves. By Julian Lowell Coolidge. Oxford, Clarendon Press, 1931. xxiv+513 pp. $10.00.

Since the appearance of Salmon's treatise nearly three quarters of a century ago, the English language has contributed but two books on this subject, both of them elementary and conservative. The German quota was well up to date at the time of its appearance, but each representative is now out of date; it remained for Italy to enrich the literature by including the latest advances in a number of recent fundamental works.

It is therefore with a feeling of pride and of satisfaction that we welcome another volume which will be of great practical value not only to English-speaking readers, but to everyone who desires a comprehensive orientation in this fundamental theory. Although all the topics considered in the earlier books are discussed in the present volume, the plan is different from that of any other treatment. A great many, perhaps too many, points of view are introduced and discussed, though some are treated much more adequately than others. The expansion of the field during the last half-century has been simply appalling; it is impossible to prepare an all-inclusive view, and there is no criterion as to what should be retained, and what omitted.

The method is algebraic, and the most important concept is that of correspondence. A chapter on the algebra of polynomials precedes the subject proper, which begins with the conditions which determine a curve, a singular point, a tangent; and almost immediately one meets Noether's famous $Af+B\phi$ theorem. The pace is brisk, but each step is convincing. The next short chapter on asymptotes and singular points is less satisfactory. As the author clearly shows in a later chapter, the only adequate tool for studying either is by means of series, without any use of derivatives. In complicated cases the equations connecting derivatives soon become too cumbersome to be of any use.

A careful and detailed analysis of the real circuits of a curve is made, with elaborate drawings. These and a few simple ones on Newton's polygon are the only figures in the book. An attempt is made to give a mathematical meaning to the phrase "in general," but the attitude concerning it is very elastic. Sometimes it is set in quotation marks, sometimes italicized, but only infrequently is the content of the defining restriction explained. The case is not helped by using the expression "usually" without connotation.

The chapter on invariants includes an introduction to the Clebsch-Aronhold symbolism for forms in n variables. I believe it is far too brief and abrupt. Apart from its use in the transcendental treatment and for the formal part of invariants, I feel that its utility is questionable. In fact, the author's reference to it in his preface strengthens my point of view.

But still more debatable is the use of the partial derivative as to the number 1. I have examined the numerous cases in which it is used, and I cannot believe that it is justified in any one of them. In fact, the use of homogeneous and of non-homogeneous coordinates appears to be inconsistent. In the discussion

of dual properties, point coordinates are treated as non-homogeneous and their corresponding line coordinates are homogeneous. Cartesian and projective systems are frequently confused.

The Cayley-Brill theory of correspondence is introduced early, and is used as a tool throughout the work. It seems to me unfortunate to use the name of Chasles to designate this fundamental theorem (Chasles-Cayley-Brill). Though historical details are not numerous, a footnote in this connection cites the essay in which it is shown that the names of two others are much more entitled to recognition than Chasles.

Chapter X of Book I, on metrical properties of curves, contains hardly a reference less than half a century old; many of these refer to books in which the theorems were already old at that time. This is the only chapter in the book that should have been omitted. Nothing in it is made use of later except a few elementary properties that could be explained in a few lines when needed.

It seems like reaching a new world when we strike Book II, on singular points. The treatments of quadratic inversion, expansion in series, and clustering singularities are excellent. The discussion of characteristic exponents and their inherent part in the given singularity is rapid but clear. The fundamental concept of satellite points is too briefly treated and rather too abstract. A schematic figure at this point would have spoken volumes. All this is done before adjoints are treated as such. The essence of the Brill-Noether theory, culminating with the Riemann-Roch theorem, can now proceed smoothly and naturally. A few repetitions appear. Thus the formula for the number of $(r+1)$-fold points of a g_n^r is given in exactly the same form on pages 133, 246, and 280.

A short chapter on abelian integrals precedes the concept of genus according to Weierstrass and that of moduli, the latter leading to the curve of lowest order with given genus and arbitrary moduli. The conditions under which further reduction can be made are not determined. After the general theorem of page 300, that of page 302 is curious. A lower order than that given is always possible under the same premises by projecting the canonical curve from any S_{p-3} meeting it in $p-3$ points into S_2. A very short chapter on special types of curves, including elliptic, hyperelliptic, polygonal, and reducible ones, is followed by one on non-linear series, preparatory to the use of abelian integrals and theta functions in the study of correspondences. Parts here are crowded and much too condensed, but the chapter on correspondences is more satisfactory. The next chapter frankly asserts that it is only a sketch as an introduction to the concept of rationalization. For example, complex multiplication is limited to $p=1$ and is disposed of in a dozen lines. The short chapter on rational curves is almost entirely formal; it starts from properties of the binary form, and confines the discussion of curves to a few things that fit this scheme. Two chapters on postulation and transformation of linear systems of curves are instructive and valuable. The proofs are frequently applicable to what the author calls the general case, but without pointing out wherein the restrictions lie. This is also true of the following chapter, which does not pretend to be complete.

In connection with the statement concerning the differences held (at one time) between Severi and Study on the validity of results obtained by the methods of enumerative geometry, it may be of interest to disclose that Study

later wrote Severi, explicitly stating that he wished to declare that the results obtained by Severi were correct and the methods completely rigorous. I do not know that this letter was ever made public.

The last three chapters are on Cremona transformations. It is a rapid survey of the whole field, too brief to be accurate or complete. But, on the whole, it is an excellent introduction to the subject. With the exception of less than two pages, this could have been put much earlier, and would have avoided unnecessary restrictions in theorems and in the point of view. A great many references are given, usually to the sources, but occasionally to the details of a discussion of which the knowledge is assumed. At times this works a hardship; a few such are used that should not be assumed to be accessible to the reader. In a few instances, important contributions had been overlooked; for example, the essays of Field on topology, that of Nichols on jacobians, and some on existence theorems of curves with given singularities.

The typographical work is excellent and the proof reading has been well done. Most of the errors noticed would not cause confusion.

On page 272 zero and infinity are at odds; on page 300 there are a number of errors, not all purely typographical. In the references, the spelling of the name of the institution at Palermo and of other Italian names is sometimes faulty, but the citations are otherwise almost without exception correct. Throughout the book the method of referring to theorems already established is confusing in the use of capital letters. The citations made are collected with full references at the end of the volume, which is also provided with an index.

Although this review contains some frank criticisms, this book is nevertheless a valuable addition to the literature on algebraic curves.

VIRGIL SNYDER

SHORTER NOTICES

Introduction à l'Étude de la Mécanique Ondulatoire. By Louis de Broglie. Paris, Hermann, 1930. xvi+292 pp.

Recueil d'Exposés sur les Ondes et Corpuscules. By Louis de Broglie. Paris, Hermann, 1930. 80 pp.

In these two books the reader will find set forth in lucid form the basic ideas of the new wave mechanics as developed by the author during the period since his epoch-making dissertation of 1924. The German translation of the first volume (which curiously enough appeared in 1929) has already been thoroughly and ably reviewed in this Bulletin by Professor R. D. Carmichael (vol. 36 (1930), p. 459) and hence further notice is unnecessary here.

The second and shorter of the two works is a collection of five rather general, non-mathematical articles and lectures written or delivered by the author during the years 1927–1929, all bearing to a considerable extent on the new wave conception of matter. The first is a historical essay on the work of Fresnel and its significance for modern physics. Written in 1927, it is peculiarly appropriate, since that year marked the one hundredth anniversary of Fresnel's premature death at the very time when his brilliant intellect seemed destined to solve completely the problems of light. The essay is introduced by an admirably succinct account of the historical development of optics to the early part of the nineteenth century followed by a recital of the way in which in the short space of twelve years Fresnel was able to place the wave theory of light on a firm foundation. The difficulties with the wave theory are then discussed together with the rise of the quantum theory with its emphasis on the particle idea. Though the physical basis of Fresnel's theory has long been seen to be untenable, the mathematical treatment of wave motion which he and Hamilton carried through has been of the greatest assistance in the development of the new theory of the past decade.

The second paper, on waves and corpuscles in experimental physics, was originally delivered as an address before the British Association at the Glasgow meeting in 1928. In it de Broglie retraces briefly the chief features of his wave mechanics and their experimental verification by the beautiful researches of Davisson and Germer, G. P. Thomson, and others. This is followed by a discussion of the precise physical interpretations which have been given to the matter wave by various workers, including Schrödinger and de Broglie himself. An unusually clear presentation of the Bohr-Heisenberg point of view and the uncertainty principle closes the essay.

The third paper, on the recent crisis of wave optics, is based on a lecture delivered in 1929 before the Conservatoire des Arts et Métiers, and takes up in conversational style the difficulties encountered in reconciling the wave and particle conceptions in optics. The fourth likewise is a short popular article on the interference of electron waves.

The last article, on determinism and causality in contemporary physics, is in many respects the most interesting in the collection since it touches on the

fascinating border region between science and philosophy. Beginning with the celebrated statement of physical determinism by Laplace, the author traces the development of the causal idea in physics up to the time when the quantum mechanics began to render its previous strict interpretation doubtful. There is again a brief survey of the fundamental ideas of wave mechanics followed by a detailed description of the content of the Uncertainty Principle, leading to the viewpoint that all that the new mechanics determines from the solutions of its differential equations and their boundary conditions is the *probability* of future events. As long as one remains on the macroscopic level (large scale phenomena) this situation reduces for practical purposes to the classical causality. But on the microscopic level (small scale phenomena) the new view will, if adopted in its present form, affect profoundly the whole course of future physical theory. It is for this reason that the new conception should be widely discussed from every conceivable angle, and every attempt like that of de Broglie to present its essential ideas in simple form is extremely welcome.

R. B. LINDSAY

Funktionentheorie. By K. Knopp. Part I, fourth edition (Sammlung Göschen, vol. 668). Berlin and Leipzig, de Gruyter, 1930. 140 pp.

This is a new edition of the first of the author's two little volumes on the subject. Under the four heads Fundamental Notions, Integral Theorems, Series, and Singularities, its eleven chapters cover much of the groundwork of the theory. A surprising amount of material is compressed into its pages. Definitions are accurately given, and many brief illustrations are introduced to clarify essential ideas. About seventy theorems are proved. The proofs are stripped of excess verbiage, but there is no sacrifice of completeness or rigor. Despite the compactness of the style, or perhaps due to it, the proofs are quite readable. Because of its emphasis on the essentials of the theory, the book would be a useful companion to the regular text in the hands of the beginner.

L. R. FORD

Projektive Geometrie. By Dr. Ludwig Bieberbach. Leipzig-Berlin, Teubner, 1931. iv+190 pp. +45 fig.

This is volume 30 of Teubner's Mathematische Leitfäden and is a continuation of the author's treatise on analytic geometry which appeared in the same collection and which gives an analytic treatment of elementary projective geometry. It is not an axiomatic exposition of the subject, although the importance of axiomatic argumentation is emphasized and utilized. From the fact that Professor Bieberbach is a noted analyst it may be expected that the rigor of analytic reasoning is manifest throughout. See for example the proof on pages 53–57 that the functions $\rho x_i' = F_i(x_1, x_2, x_3)$, $(i = 1, 2, 3)$, which transform lines into lines throughout the projective plane, are linear.

One welcome feature of the little treatise is the inclusion of some interesting propositions on the geometry of the triangle as features of metric specialization.

Altogether, Professor Bieberbach's *Projective Geometry* is an excellent introduction to the subject, as an analyst conceives it, and contains many valuable features.

ARNOLD EMCH

A Course of Analysis. By E. G. Phillips. Cambridge University Press, 1930. vii+361 pp.

This book is based on the first part of a course of lectures on analysis given by the author to mathematical honors students in the University College of North Wales. The presentation follows the traditional English manner, clear, rich in content, and compact. The topics considered cover a wide range and their choice and arrangement are well calculated to arouse and hold the interest of the student. The work is intended for students of moderate mathematical maturity who have been prepared by a standard course in calculus.

The first chapter is devoted to an outline of the logical foundations of analysis and the structure of the number system. Chapters on sequences and limits and the theory of continuous functions are followed by an exposition of the elements of the differential calculus. Here the distinction between derivable and differentiable is introduced. This finds an important application later in the chapter on functions of several variables. A brief introductory chapter on series designed to provide a basis for later developments is followed by an interesting chapter on the inequalities of Hölder, Minkowski, Jensen and others. Inequalities are treated throughout the work thoroughly and elegantly, a feature of special value.

The theory of integration is limited to the Riemann integral. This limitation can hardly be avoided in an introductory volume. The distinction between a primitive and an indefinite Riemann integral is carefully illustrated. Infinite integrals are briefly treated.

After extending results previously obtained to functions of several variables the author presents in the tenth chapter a readable discussion of implicit functions with an application to the theory of the exponential function and an adequate treatment of transformations and jacobians. Chapters on multiple integration with particular reference to classical formulas of mathematical physics follow. The work concludes with a short chapter on power series and functions defined by power series, with an application to the theory of the circular functions.

Lists of exercises at the chapter ends contribute materially to the value of the work as a text. As many of the exercises are beyond the power of average students, it would be beneficial to have additional exercises for practice.

In a work of this scope and brevity it is difficult to secure perfect balance. Some subjects such as infinite integrals are too briefly treated, others, for example the integration of non-uniformly convergent sequences, receive more attention than is desirable in an introductory course. While misprints are rare, the author has frequently failed in his effort to present the student with an accurate exposition. Gehman (American Mathematical Monthly, vol. 38, pp. 166–167) has pointed out a number of slips and commented on the frequent errors in the statement of theorems, particularly mean-value theorems. He calls attention to a fallacious proof on page 32 of the theorem of Weierstrass. Another notable error occurs in the proof of Theorem 3, page 344; it invalidates the demonstration of Abel's theorem on power series. In theorems involving transformations of multiple integrals, Phillips often fails to assume that the jacobian does not change sign throughout the region of integration.

In addition to the lapses in accuracy mentioned, it seems proper to call attention to the fact that this book shares with other introductory works on analysis the fault of creating an illusion of simplicity, facility, and even completeness. The addition of a thoroughgoing treatment of some fundamental topic as an example of the nature of a rigorous discussion would improve the work.

It is usual for authors of works of this sort to introduce the elementary transcendental functions for purposes of illustration in advance of their theoretical treatment. While this method undoubtedly adds to the interest of the discussion it seems likely that the student will eventually fail to distinguish clearly between the material which is logically presented and that which is illustrative. This can be avoided by an obvious rearrangement of subject-matter. This rearrangement is much to be preferred if the reader's interest can be sustained.

E. W. CHITTENDEN

Wahrscheinlichkeitsrechnung und ihre Anwendung in der Statistik und Theoretischen Physik. By Richard von Mises. Leipzig and Wien, Franz Deuticke, 1931. 574 pp.

Aside from its usefulness in technical applications, the theory of probability has recently taken on a significance secondary to that of very few mathematical issues. Philosophers have come to study it with a view to investigate its logical implications, and physicists have recently been accustomed to regarding it as the foundation of their science. Their acceptance of probability doctrines has frequently been uncritical, and it is safe to say that many difficulties in connection with the present status of quantum dynamics are consequences of an incomplete understanding of the basic probability postulates. In the midst of this situation the appearance of a book like that of von Mises is indeed a fortunate event.

It is well known that the author's views concerning the axioms of the probability theory are specific and exclusive, that they are disputed in several quarters. Nevertheless, in the opinion of the reviewer their value in applications, such as those made in the exact sciences, is unique, and it is gratifying to observe that, in this book, von Mises' frequency theory has been carried through consistently in a manner that is detailed and complete.

The book is the first volume of a series of treatises on applied mathematics. Continuations, dealing with hydromechanics, practical analysis, and applied geometry, are announced in the author's preface. The present volume contains four principal sections; the first two are concerned with the theoretical elements of the theory, the third is an application of probability to statistics and the theory of errors, the last discusses in a lucid fashion the foundation of physical statistics. Numerous problems of historical and practical interest are treated in the text, and to each section is appended a set of problems to be solved by the reader. The book is intended as a textbook. In conformity with this aim, the author has succeeded in making his exposition clear and concise. The treatment is more extended than is customary in American texts, but the greater length makes for easy reading. Indeed its reading is distinctly less difficult than that of most similar treatises, in spite of the complete logical rigor of the

presentation. This fact is particularly noteworthy in view of the circumstance that the development is built on axioms which are, in a sense, novel and unconventional.

The enormous wealth of the subject matter makes it impossible for the reviewer to pass individual judgment on detailed matters. As would naturally be expected, the author's illustrations center largely about his own researches. But this is felt to be no loss, since they are so extensive and varied as to impress no bias upon the general discourse.

Readers who are unwilling to work through the total volume, but desire information about specific topics, may use the book with considerable profit. The different sections, while logically and pedagogically related, can be understood fairly well without a thorough study of the preceding ones, and the alphabetical index is complete.

The printing is agreeable to the eye, and the number of misprints is remarkably small.

Criticism can be levelled at most against the author's philosophical stand, but this is likely to be quite pleasing to scientists. Only toward the end of the book, where the author deals excellently with the ergoden hypothesis, and with transition probabilities, the physicist feels that it would have been extremely desirable if the analysis had been extended to the more troubling problems encountered at present in quantum physics.

HENRY MARGENAU

Infinite Series. By Tomlinson Fort. Oxford University Press, 1930. iv+253 pp.

Professor Fort has written an excellent book of the type that he set out to write. The proofs are in general clean cut and clear, and it is evident that the work has been prepared with much care and thought.

The book is less comprehensive in scope and thus better adapted to the needs of the beginner than the well known treatises of Bromwich and Knopp. The exercises are sufficiently numerous and are well selected, thus adding to the value of the volume from the point of view of instruction. It is a matter of regret, in the opinion of the reviewer, that these advantages are somewhat counterbalanced by the form of exposition that has been chosen by the author. All results appear as numbered theorems with little or no suggestion as to their relative importance. It would require a considerable amount of perspicacity on the part of the unsophisticated reader to locate without assistance what might be termed the central features of the theory. For example, the fundamental necessary and sufficient condition for the convergence of a sequence appears as Theorems 15 and 16 (necessary and sufficient condition, respectively), with no particular indication of its unusual importance from the theoretical standpoint. We might remark in passing that the proof of the sufficient condition is more involved than need be.

The reviewer agrees in general with the selection of material and the relative amount of space allowed for various topics. He would prefer to see some of the more recondite parts of the theory of series, such as quasi-uniform convergence and similar topics, omitted, and more space devoted to such an important type of series as Fourier series. Likewise, he would advocate a different apportionment of space among the various methods for summing divergent

series, although the total number of pages devoted to this topic (forty-five) seems to be adequate for an introductory treatise.

In the chapter on multiple series the author follows Jordan in defining convergence in such a manner as to include only absolutely convergent series. The case of conditional convergence, usually designated as convergence in the Pringsheim sense, is relegated to one of the exercises. Both definitions go back to Cauchy, although the latter seems to have been unaware of the fact that they were not co-extensive. The distinction between them was pointed out in an article by Stolz, entitled *Ueber unendliche Doppelreihen*, which appeared in 1884 in volume 24 of the Mathematische Annalen. The detailed studies of the case of conditional convergence made later by Pringsheim have caused the latter's name to be associated with this type of convergence. It is true that absolutely convergent multiple series have certain useful properties not possessed by series that are only conditionally convergent. An analogous situation occurs, however, in the case of simple series, and there seems to be no adequate reason for restricting the definition of convergence for multiple series in such a manner as to rule out conditional convergence. Such a restriction is certainly not in harmony with general usage among modern researchers in the field of multiple series.

In view of the prevailing tendency, even in elementary instruction, to stress the importance of acquiring historical background in connection with the study of mathematics, one is surprised at the scant number of references to the literature of the subject that are found in the book. The reviewer thinks that when a student has reached the point where he is ready and willing to study such a topic as infinite series, he should begin to appreciate, if he has not previously done so, that mathematics is a living and growing science. He should be made to realize that any treatise which he studies is in the main an organization of results that have been discovered by various mathematicians of greater or lesser eminence, and he should learn the names of some of them. He should feel that, in addition to the direct knowledge gained, he is acquiring an avenue of approach to a far richer supply of related results, and in some cases at least, putting himself in a position to add to the stores of knowledge in the field in question. The book under review is not designed materially to help the student to any such realization.

Having registered dissent with the author's decision as to the manner in which he chose to present his material, the reviewer hastens to add that he nevertheless regards the book as a useful addition to the available literature on infinite series.

C. N. MOORE

Leopold Kronecker's Werke. Volume 3, Part 2. Edited by K. Hensel. Leipzig, B. G. Teubner, 1931. 216 pp.

This volume completes the edition of Kronecker's mathematical papers; it contains 9 papers, including the very important series of communications on modular systems and the theory of general complex numbers. Then follow applications of this theory, and two papers on the reduction of systems of quadratic forms. Hensel has added a few explanatory footnotes and a short list of misprints.

OYSTEIN ORE

Elementary Theory of Finite Groups. By Louis Clark Mathewson, under the editorship of John Wesley Young. Boston, Houghton Mifflin, 1930. x+165 pp.

It is a fortunate fact that as mathematics progresses, large bodies of doctrine once on the outskirts of knowledge become organized in such a way that the ordinary student of college mathematics may procure a working acquaintance with them. This, for instance, has been the case with the calculus, much of the theory of equations, and projective geometry.

The theory of groups had long ago passed the stage of a set of isolated facts. It had become an organized structure. It was both inevitable and desirable that books should be written aiming at the exposition of group theory in a manner suitable for use in undergraduate classes. Dr. Mathewson has sought to do this. He has been faced with many problems, and it is to be expected that in seeking their solution he should not please everyone.

There is the problem of selection and arrangement of material. This has been admirably solved. After a few examples of groups, the elementary theory of permutation groups is given. This is followed by examples of an interesting nature, after which the general theory is renewed. There is then a chapter on abelian groups, one on abstract definitions, and one on isomorphisms and composition-series. Two chapters sketching important further developments conclude the book. The first of these on linear substitutions gives some proofs, the last, chiefly dealing with Galois theory of equations and the Lie theory, through the statements of definitions and theorems, conveys to the reader some idea of the richness of these fields.

The author of such a book must also answer two important questions. How rigorous and general should the proofs be? What type of notation should be used? One should not be dogmatic on these points. It is certainly not harsh criticism; it may even be praise to say that the author's answers to these questions differ widely from those of the reviewer. The reviewer believes that throughout our college texts too little regard for powerful and general notations and too much negligence of rigor are shown. In subjects like algebra and the calculus it may be argued that the manipulative use of the methods involved is so important that questions of rigor may, to a certain extent, be ignored. It would seem, however, that a study of group theory would be largely valuable as showing the student an example of rigorous thought seldom met with before graduate work, and as giving the student an understanding of the value of powerful concentrates in notation. It seems fair to believe that the author does not agree with this point of view. It is true that the proofs are such that a student, seeing the lack of rigor or of generality, is given enough hints to fill in the gap. The student would not, however, be led to note the lack. For instance, in proving that the number of transpositions into which a given permutation can be factored is always even or odd, a Vandermondian determinant is used without establishing the fact that it does not vanish—a point necessary to the argument. In proving that the order of a sub-group is a factor of the order of the group, the inductive step is omitted, though the form it would take has been made obvious. In the notation distinct letters are used where subscripts would seem more powerful; matrices are displayed rather than written in abbreviated form; and similar usages are followed throughout the text.

The examples are chosen with great skill.

We should all welcome such a well thought out endeavor to make the theory of groups more teachable. But as long as mathematicians remain as happily diversified as at present some will think that such a book as that under consideration has made too great sacrifices of rigor and generality, while others would condemn the books the reviewer would like to see written as too abstract and their authors as too visionary. Nothing is more disputable than taste.

M. H. INGRAHAM

The Universe in the Light of Modern Physics. By Max Planck. Translated by W. H. Johnston. New York, Norton, 1931. 118 pp. $2.00.

Planck (p. 8) insists that we are compelled to assume the existence of a world of reality behind the usual world of the senses and that progress in physics has been made only on this assumption. "But (p. 10) besides the world of sense and the real world, there is also a third world which must be carefully distinguished from these:—this is the world of Physics." The most far-reaching results of investigation (p. 15) can be obtained only by following a road leading to a goal which is theoretically unobtainable; and that goal is the apprehension of true reality: as the view of the physical world is perfected, it recedes from the world of sense and approaches the world of reality. The principal intellectual tool for such progress is (p. 65) the hypothesis whose implications are drawn and subjected to the test of experimental verification.

In relativity and quantum theory (p. 18) we have two completely new ideas, but their advent means (p. 23) reconstruction, not destruction. The revolutionary consequences of these ideas are treated at some length, especially in their bearing (p. 28) on the meaning of a particle, on the nature (pp. 39–41) of the principle of uncertainty and (p. 45) the character of waves of probability, and finally on causality, determinism, indeterminism and free will. Whatever doubt (p. 59) these new ideas may have thrown on the question whether physics is really on the right path, Planck meets with the proposition (p. 60) that theory and experience were never before so closely linked in physics and with the confident prediction (p. 112) that the parts of classical theory which have had to be discarded will be supplanted by what is sounder and more adequate.

The new physics (p. 113) will give us an insight into the secrets of the theory of knowledge. A new need has arisen for ideas from philosophy. "For this reason (p. 113) a careful study of the views and ideas of our great philosophers might prove extremely valuable." "Modern Physics (pp. 113–4) impresses us particularly with the truth of the old doctrine which teaches that there are realities existing apart from our sense-perceptions, and that there are problems and conflicts where these realities are of greater value for us than the richest treasures of the world of experience."

These and similar propositions are urged with supporting reasons. The translation appears to be well done. The typography reaches a high order of excellence.

R. D. CARMICHAEL

NOTES

The following have been appointed associate editors of the Transactions of this Society: E. P. Lane, Marston Morse, M. H. Stone.

At the New Orleans meeting of the American Association for the Advancement of Science, Professor H. H. Mitchell was elected vice-president of Section A and Professor D. R. Curtiss a member of the executive committee of the Association.

The scientific part of the program of the International Congress of Mathematicians to be held in Zurich, September 4–12, 1932, will consist of general lectures and of meetings of the Sections. The following have already agreed to deliver general lectures: Alexander, Bieberbach, H. Bohr, Carathéodory, Carleman, E. Cartan, Fubini, Fueter, Hardy, Julia, Menger, Morse, R. Nevanlinna, W. Pauli, F. Riesz, Severi, Sierpinski, Stenzel, Tschebotaröw, Valiron, Wavre, Wedderburn. The shorter papers will be presented before the following Sections (which may be further subdivided): algebra and theory of numbers, analysis, geometry, probabilities and insurance mathematics, mechanics and physics, astronomy, engineering, logic and philosophy, history, pedagogy. The program also includes plans for social functions and excursions.

The Paris Academy of Sciences announces the award of the following prizes for 1931: the Francœur prize to Jacques Herbrand, for his work in the theory of corps of numbers; the Montyon prize in mechanics to Hippolyte Parodi, for his work in the electrification of railroads and in ballistics: the Poncelet prize to Henri Chipart, for his work in mathematical physics and mechanics; the Lalande prize to Irénée Lagarde for his work relative to astronomical calculations; the Valz prize to Henri Chrétien, for his work in astronomical optics; the de Pontécoulant prize to Jean Chazy, for his work in analytical and celestial mechanics; the Bordin prize to René Garnier, for his work on the problem of Plateau; the Petit d'Ormoy prize to Gaston Julia, for his mathematical work; the Jean Reynaud prize to the late Paul Appell, for his scientific work as a whole; the Saintour prize to Henri Devaux, for his work on the properties of thin films on the surface of liquids. The Academy announces the following subject for its Bordin prize for 1933: Mathematical physics has, from the beginning, made use of two profoundly different methods for representing solutions. On the one hand, these may be represented by definite integrals depending on the data of the problem; on the other hand, these data being represented in series of some appropriate form, of which the best known is the Fourier series, it may be proposed to put the solution in an analogous form. Every element of the data influences the solution directly in the first form, while they appear only in the mass ("globalement") in methods of the second kind. The Academy proposes the search for a connection between these two categories of methods. One might, for example, study in this spirit the Fourier series (or some analogous form of series) that differ from zero only in a part of their interval of definition.

The Edison medal for 1931 has been awarded to E. W. Rice, Jr., of the General Electric Company, "for his contributions to the development of electrical systems and apparatus and his encouragement of scientific research in industry."

On September 9, 1931, a monument to Hendrik Antoon Lorentz was unveiled in the Sonsbeekpark in Arnhem, by Princess Juliana, of the Netherlands.

Professor Emile Picard, permanent secretary of the Paris Academy of Sciences, has been elected a member of the French Council of Public Instruction.

Professor H. N. Russell, of the department of astronomy of Princeton University, has been elected president of the American Philosophical Society.

At the University of Poitiers, Professor G. Bouligand has been transferred from the professorship of rational mechanics to that of the differential calculus.

Professor B. Schilling has returned to the Dresden Technical School as associate professor of mathematics, after having served for two years in the department of mathematics at the University of Santiago, Chile.

Dr. W. G. Bickley has been appointed reader in mathematics at the City and Guilds College, Imperial College, University of London.

Dr. G. F. J. Temple has been appointed reader in mathematics at the Royal College of Science, Imperial College, University of London.

Professor Wilhelm Blaschke, of the University of Hamburg, in 1931 Visiting Lecturer of the American Mathematical Society, lectured in February, 1932, at the Universities of Annamalai and Calcutta, and will lecture in April at the Universities of Tokyo and Sendai, in May at Stanford University, and during the summer session at the University of Chicago. His subjects will be differential geometry, topology, and continuous groups.

Professor Albert Einstein has been appointed Rouse Ball lecturer at the University of Cambridge for the year 1931–32. Professor Einstein has been in residence at the California Institute of Technology and the Mount Wilson Observatory during January and February, 1932.

Dr. Willem de Sitter, director of the Astronomical Observatory of the University of Leiden, delivered a series of lectures in January, 1932, at the University of California, on *The astronomical aspects of the theory of relativity*, and *The system of astronomical constants*. He has also lectured recently at several other American universities.

Professor T. R. Hollcroft, of Wells College, delivered an address entitled *Algebraic curves and surfaces with assigned singularities* before the mathematical section of the University of Durham Philosophical Society, at Armstrong College, Newcastle, on November 27, 1931.

Professor Oswald Veblen, of Princeton University, is on leave of absence during the second semester of the current academic year. On December 18, he lectured at Duke University on *Spinor analysis;* on January 8, 11, and 12, he lectured at the Rice Institute on the topics *The modern approach to elementary geometry, Analysis situs,* and *The new differential geometry;* and on January 25, he lectured at The University of California at Los Angeles on *Projective relativity.* In February he will go to Europe, and will lecture at the universities of Goettingen, Hamburg, and Berlin.

A professorship of mathematics has been established at Yale University in honor of Josiah Willard Gibbs. Professor E. W. Brown, previously Sterling professor of mathematics at the University, has been made the first incumbent of this new professorship.

Assistant Professor J. A. Nyswander, of the University of Michigan, has been promoted to an associate professorship.

Professor Warren Weaver, Chairman of the Department of Mathematics at the University of Wisconsin, was appointed Director of the Division of Natural Sciences of the Rockefeller Foundation on February 1, 1932. Professor M. H. Ingraham will act as Chairman of the Department in his stead.

Mr. D. E. Whitford has been promoted to an assistant professorship at the Brooklyn Polytechnic Institute.

Professor Maurice Alliaume, of Brussels, editor of the mathematical section of the Annales de la Société Scientifique de Bruxelles, is dead.

Gabriel Koenigs, professor of physical and experimental mechanics at the Sorbonne, died in October, 1931, at the age of seventy-three.

Gabriele Torelli, professor emeritus of mathematics at the University of Naples, died November 7, 1931.

The announcement has been received of the death of Professor Heinrich Wieleitner, Oberstudiendirektor am neuen Realgymnasium und Honorarprofessor an der Universitat, München, on December 27, 1931. Professor Wieleitner was well known to all students in the history of mathematics. He was a serious scholar and a clear and accurate writer. His loss will be felt in this country as well as in Munich, where he worked for so many years.

The death is reported of Alexander Knisely, of Columbia City, Indiana. He had been a member of the American Mathematical Society since 1891.

Miss Rose B. Wood, of Greenville Woman's College, a member of the American Mathematical Society, died July 28, 1931.

Professor J. W. Young, of Dartmouth College, died February 17, 1932, at the age of fifty-two. Professor Young had been a member of the American Mathematical Society since 1902, and an editor of the Bulletin from 1907 to 1925. A more adequate notice will appear in an early issue.

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THIS SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

58. Dr. L. S. Kennison: *Conformal transformations in function space.*

Conformal transformations of the space L_2 are defined and shown to form a group. The following analog of Liouville's theorem is proved: All conformal transformations in function space are composed of translations, rotations, transformations of similitude, and inversions. (Received January 19, 1932.)

59. Dr. L. S. Kennison: *Note on homogeneous functionals.*

The analog for functionals of Euler's theorem for homogeneous functions of n variables was proved by E. Freda in 1915. This note contains a proof of this theorem for a more general case, also a proof of the theorem, not proved by Freda, that if $F[\lambda\phi(x)] = K(\lambda)F[\phi(x)]$, then $K(\lambda)$ is a power of λ. (Received January 19, 1932.)

60. Professor J. H. Roberts: *Concerning spaces which are uniordered relative to systems of closed and compact point sets.*

G. T. Whyburn has raised the following question (see Fundamenta Mathematicae, vol. 16, p. 170): *Can every separable metric space S which is uniordered relative to a system Z be transformed by a biunivalued and continuous transformation into a separable metric space S^* in which every point P^* is contained in arbitrarily small neighborhoods with boundaries belonging to Z?* The space S is uniordered relative to Z if for every point P there exists a monotonic sequence of domains $U_1, U_2, \cdots$, containing P, each bounded by an element of Z, and such that $P = \Pi(\overline{U}_i)$. By a "system Z" is understood a collection Z of closed point sets such that every closed subset of an element of Z is an element of Z, and the sum of every two elements of Z is an element of Z. An example shows that the answer to Whyburn's question is in the negative. However, it is proved that with the additional hypothesis that the elements of Z be *compact* the answer is in the affirmative. (Received January 15, 1932.)

61. Professor J. L. Walsh: *An expansion of meromorphic functions.*

If $f(z)$ is a meromorphic function (that is, analytic, except for poles, at every finite point of the plane) whose poles are included in the sequence $\alpha_1, \alpha_2, \cdots$, and if the points $\beta_1, \beta_2, \cdots$ are distinct from the α_n and uniformly limited, then the formal expansion of $f(z)$, $f(z) = a_0 + a_1(z-\beta_1)/(z-\alpha_1) + a_2(z-\beta_1)(z-\beta_2)/((z-\alpha_1)(z-\alpha_2)) + \cdots$, found by interpolation in the points β_n converges to $f(z)$ for all finite values of z except the α_n. In particular, if $\beta_1 = 0$, $\beta_{n+1} = 1/\bar{\alpha}_n$, and if $f(z)$ is analytic for $|z| \leqq 1$, then the sum of the first $n+1$ terms of this series is the function $(a_{0n}z^n + a_{1n}z^{n-1} + \cdots + a_{nn})/((z-\alpha_1)(z-\alpha_2)\cdots(z-\alpha_n))$ of best approximation to $f(z)$ on the unit circle in the sense of least squares. (Received January 15, 1932.)

62. Professor J. L. Walsh: *Interpolation and functions analytic interior to the unit circle.*

If the product $\Pi|\beta_n|$ converges, $|\beta_n| < 1$, and if we have $f(z) = (1/(2\pi i)) \cdot \int_C (f_1(t)/(t-z))\, dt$, $C: |z| = 1$, where $f_1(z)$ is integrable together with its square on C, then the formal expansion of $f(z)$ in a series $f(z) \sim a_0 + a_1 z/(1-\beta_2 z) + a_2 z(z-\beta_2)/((1-\beta_2 z)(1-\bar{\beta}_3 z)) + \cdots$, found by interpolation in the points $\beta_1 = 0, \beta_2, \beta_3, \cdots$, or (what is equivalent) by expanding $f_1(z)$ formally on C in this series of orthogonal functions, converges for $|z| < 1$, uniformly for $|z| \leqq r < 1$. The function $\phi(z)$ represented coincides with $f(z)$ in the points β_n and can be expressed by $\phi(z) = (1/(2\pi i)) \int_C (\phi_1(t)/(t-z))\, dt$, where $\phi_1(t)$ is integrable together with its square on C. (Received January 16, 1932.)

63. Professor J. L. Walsh: *Note on the degree of convergence of sequences of analytic functions.*

If the sequence of functions $f_n(z)$, analytic in a region C, converges uniformly in C to the function $f(z)$ with a certain degree of convergence, and if the function $f_n(z) - f(z)$ is known to have a certain number of zeros at specified points interior to C, then Jensen's theorem yields a result on the degree of convergence of $f_n(z)$ in an arbitrary region interior to C. This remark is particularly applied to the study of series of the form $a_0 + a_1(z-\beta_1)/(z-\alpha_1) + a_2(z-\beta_1)(z-\beta_2)/((z-\alpha_1)(z-\alpha_2)) + \cdots$. (Received January 15, 1932.)

64. Professor J. L. Walsh: *On the overconvergence of sequences of rational functions.*

This paper considers more general situations on the overconvergence of sequences of rational functions than have hitherto been studied, and indeed seems to prove the most general theorem of its kind. (Received January 15, 1932.)

65. Professor J. L. Walsh: *On polynomial interpolation to analytic functions with singularities.*

Let $f(z)$ be continuous on C: $|z| = 1$, and let $p_n(z)$ denote the (unique) polynomial of degree n which coincides with $f(z)$ in the $(n+1)$st roots of unity. Then we have $\lim_{n\to\infty} p_n(z) = (1/(2\pi i)) \int_C (f(t)/(t-z))\, dt$ uniformly for $|z| \leqq r < 1$. (Received January 15, 1932.)

66. Dr. G. T. Whyburn: *A certain transformation on metric spaces.*

In this paper a study is made of the transformation T effected on a connected and locally connected metric space M by changing all distances $\rho(x, y)$ into distances $\rho^*(x, y) = \min [\delta(C)]$, where C is any connected subset of M containing $x+y$. It is shown, among other results, that T is a homeomorphism whose inverse is uniformly continuous and which leaves invariant diameters of connected sets and hence also Property S. Furthermore, $T(M)$ is uniformly locally connected. The results found are applied to show that if R is any simply connected plane region having Property S with boundary B, then the boundary J of $T(R)$ is a simple closed curve; and since the inverse of T is uniformly continuous, it can be extended in one and only one way to give a continuous transformation W of J into B. Furthermore, W is a minimal mapping, from the standpoint of multiplicity, of J onto B. The multiplicity under W of any point p of B is equal to the number of components of $B-p$ when this number is finite, and is infinite when this number is infinite. (Received January 8,1932.)

67. Professor E. B. Stouffer: *A geometrical determination of the canonical quadric of Wilczynski.*

Canonical developments for the equation of a curved surface take their simplest forms when the vertices of the associated tetrahedrons of reference lie on the canonical quadric of Wilczynski. In the present paper this quadric is located geometrically in a natural and simple manner by means of its intersection with the axis of Čech. (Received January 20, 1932.)

68. Professors A. A. Albert and Helmut Hasse: *A determination of all normal division algebras over an algebraic number field.*

In a p-adic arithmetic treatment, H. Hasse has reduced the problem of proving that every normal division algebra over an algebraic number field F of finite degree is a cyclic (Dickson) algebra to the question of the validity of the following theorem: If a normal division algebra A of degree M (order m^2) over F splits everywhere, then $m=1$. Hasse also proved that this latter theorem is true if we assume that A is cyclic of degree unity, or a prime p. It is shown now how the above theorem is an immediate consequence of Hasse's arithmetic results and already published theorems of A. A. Albert. Also a new proof is given following the ideas of Albert's proof but with simplifications due to the omission of extraneous material. (Received January 20, 1932.)

69. Professor A. A. Albert: *On normal simple algebras.*

The author's theorems on the direct product of a normal division algebra and an algebraic field (Transactions of this Society, vol. 33 (1931), pp. 690–711) have proved to be very powerful tools for research in the theory of linear algebras. In the present paper the author throws new light on the meaning of the former theorems in a study of the properties of a subfield K of any normal simple algebra A relative to the division algebra component of A. Finally the results obtained are used to prove the validity of a conjecture of L. E.

Dickson (Transactions of this Society, vol. 28 (1926), pp. 227, 228) as to a necessary and sufficient condition that a normal simple algebra be a division algebra. (Received January 20, 1932.)

70. Professor E. P. Lane: *Surfaces and curvilinear congruences.*

The purpose of this paper is to begin the study of the projective differential geometry of the configuration composed of a surface and a curvilinear congruence in ordinary space, the points of the surface and the curves of the congruence being in one-to-one correspondence. This generalization from rectilinear to curvilinear congruences possesses interest because it not only contains many of the results of recent literature asspecial cases but also leads to new and more general results. Congruences of plane curves, one of which lies in each tangent plane of the surface, receive particular attention, and still more specially such congruences of conics and plane cubic curves are studied. (Received January 29, 1932.)

71. Professor E. B. Stouffer: *On the contact of two space curves.*

Associated with two space curves which have contact of order n at a point there are a principal plane discovered by Halphen and in general a principal line and a principal point discovered by Bompiani. In the present paper all three of these elements are obtained by a simple process involving only a linear transformation. The method may be extended to obtain similar results in hyper-space. (Received February 10, 1932.)

72. Dr. R. S. Burington: *A classification of quadrics in affine N-space by means of arithmetic invariants.*

The classification of real conics under the euclidean group has been discussed by MacDuffee, Paradiso, and Franklin, with the use of algebraic invariants (American Mathematical Monthly, vol. 33, pp. 243–252, pp. 406–418; vol. 34, pp. 453–467). In some of their euclidean canonical forms there are infinitely many quadrics, all of which are equivalent under the affine group, and not equivalent to a quadric of any other canonical form. Thus the problem of the separation into types is a problem in real affine geometry. It is the purpose of this paper to show that the matrix of the quadric has four arithmetic invariants under the real affine group which are sufficient to give a complete separation of the quadrics into types. The types obtained by this means coincide exactly with those previously obtained, and the labor in applying the theory to a given quadric is slight. The types of real quadrics for $n=2, 3,$ and 4 are listed in detail. (Received February 11, 1932.)

73. Professor A. A. Albert: *A construction of non-cyclic normal division algebras.*

A construction is given of two generalized quaternion algebras B and C over a function field $F(u, v)$ of all rational functions with coefficients in a real

field F of two independent marks u and v. It is shown that the algebra $A=B\times C$ is a normal division algebra of order sixteen and is not a cyclic (Dickson) algebra. This is the first example of non-cyclic algebras in the literature, and the very important question as to their existence is finally settled. (Received February 20, 1932.)

74. Professor W. D. Baten: *Sampling from many parent populations.*

Out of the parent population A_i, consisting of n_i variates, r_i variates are taken at random. The sample considered in this article consists of r_1 variates from parent A_1, r_2 variates from parent A_2, $\cdots$, and r_s variates from parent A_s. All possible sample means are obtained. The distribution of these sample means is examined from the nature of the standardized moments when the number of variates in each parent is finite, when the number of variates approaches infinity, and finally when the numbers of variates taken from each parent approach infinity. General formulas are given for determining any moment by use of the Carver polynomials. This distribution of sample means is shown to approach the normal distribution when the parents contain an infinite number of variates and the number of variates taken at random from each parent approaches infinity, regardless of the nature of the parent populations. (Received February 24, 1932.)

75. Professor H. S. Wall: *On the Padé table for a power series having a corresponding continued fraction in which the coefficients have limiting values.*

Let $P(z)$ have a corresponding continued fraction $b_1/(1+(b_2z/1+(b_3z/1+\cdots)))$ in which $\lim_n b_n=b$. Then if $b=0$ and $\lim_n(b_n/b_{n-1})=r>0$, every diagonal file of the Padé table converges to one and the same meromorphic limit. If $b\neq 0$ the files converge to a common limit over the entire plane except along the whole or a part of that segment of the real axis from $x=-\frac{1}{4}b$ to $x=\infty$ which does not contain the origin, and except possibly at certain isolated points. Within the plane so cut the limit is holomorphic except at these isolated points, which are poles. (Received March 1, 1932.)

76. Mr. Rufus Oldenburger: *Canonical binary trilinear forms.*

E. Schwartz obtained binary trilinear forms canonical under transformations in the complex field. These forms are here obtained directly from the theory of pairs of binary bilinear forms, and are characterized by two arithmetic invariants one of which is the generalization of ordinary matrix rank. (Received March 5, 1932.)

77. Professor C. C. Camp: *A new method for finding the numerical sum of an infinite series.*

$\sum_{n=1}^{\infty} u_n=s$, where u_n is a positive quantity, can be evaluated by defining u_n for all real $n\geq 1$ as in the Cauchy integral test, adding the first n terms, and estimating the remainder R_n by formula (1): $\int_{n+1}^{\infty}u_n dn+\frac{1}{2}u_{n+1}<R_n<\int_n^{\infty}u_n dn-k_n u_n$,

where $k_n = (\int_n^{n+2} u_n dn - u_{n+1} - u_{n+2})/(u_n - u_{n+2})$. We take u_n so that its derivative will approach zero monotonically. The upper bound is closer to R_n than the lower, the ratio of the deviations being about $1:p$ for $u_n = n^{-p}$. Accordingly, for $n=20$ one estimates s correct to 8 decimals for $p=2$. By using (1) for $n=5$ and $n=9$ for the series (2): $u_n = (n^{-5/2} + n^{-7/2}/4 + 21\ n^{-9/2}/128)/16(\pi n)^{1/2}$, which arises in the integration of $\int_0^1 (1-u^8)^{1/2}\, du$, one obtains upper bounds a_1, a_2 for s; and lower bounds b_1, b_2. Straight line extrapolation $t = b_2 + (b_2 - b_1)\Delta_2/(\Delta_1 - \Delta_2)$, where $\Delta_i = a_i - b_i$, $i=1, 2$, yields the sum correct to 7 decimals. Alternating series are converted into positive series by pairing terms. Stirling's series were used to obtain the asymptotic formula (2) and Simpson's rule to compute k_n. In summing Maclaurin's series for $\sin^{-1} 1$ similarly, $n=8, 12$ gave t with an error $< u_{13}/6000$. (Received March 5, 1932.)

78. Dr. L. M. Blumenthal: *Concerning regular pseudo d-cyclic sets.*

In a paper now in press (*A complete characterization of proper pseudo d-cyclic sets of points*, American Journal of Mathematics) the author has characterized those pseudo d-cyclic sets that (1) contain no convex tripod, and (2) contain no pseudo d-cyclic quadruples that are pseudo-linear. Such sets were called proper. The purpose of this paper is to characterize pseudo d-cyclic sets that contain no convex tripods, the second condition above not being retained. Such sets are called *regular*. The principal theorem of the paper proves that such sets are equilateral provided that they contain more than four points. (Received March 5, 1932.)

79. Mr. J. H. Kusner: *On continuous curves with cyclic connection of higher order.*

J. R. Kline has raised the question whether, for $n>2$, there exists a continuous curve C such that every two points of it are joined in C by exactly n arcs which are mutually exclusive except for end points. It is here shown that for $n = 3$ no such curve exists. If, for any $n>2$, such a curve existed, it would have the property that every three of its points lie on a simple closed curve in it. The question arises whether there exist regular curves which have this property. The Sierpinski triangle curve is obviously of this type, and it is shown that any finite number of its points lie on a simple closed curve in it. A continuous curve C such that any n points of it lie on a simple closed curve in C is said to be n-cyclicly connected, in extension of the notion of cyclic connection introduced by Whyburn in 1927. Necessary and sufficient conditions for a continuous curve to be 3-cyclicly connected are derived, and some of the properties of such curves are developed. (Received March 5, 1932.)

80. Dr. Leo Zippin (National Research Fellow): *Irreducible continuous curves.*

It is proved that in order that a self-compact one-dimensional subset K of a generalized continuous curve (quasi-Peanian space) belong to a subcontinuous curve C'' of C which is *irreducible about* K, it is necessary and sufficient

that (1) the components of K be continuous curves of points; (2) for every n, the components of K of diameter $1/n$ be in finite number. As special cases: whenever the curves of K belong to the class of acyclic, regular, perfect, or rational curves respectively, then C'' belongs to the same class. Ours is essentially an extension of an old plane theorem of Gehman. The restriction upon the dimension of K is occasioned by our proof, and is probably not required by the theorem. (Received March 3, 1932.)

81. Dr. A. B. Brown: *On Morse's duality relations for manifolds.*

Duality relations have been proved by Marston Morse (National Academy Proceedings, vol. 13 (1927), pp. 813–817) for cases which may be described, except for certain analytic restrictions on the configurations involved, as that involving the Betti numbers of a manifold with regular boundary and those of its boundary, and that involving the Betti numbers of a sub-complex of an absolute n-manifold, and the Betti numbers of the residual set. In the present paper, with the aid of duality relations of Alexander and Lefschetz, we establish the duality relations without analytic restrictions, and in the second case remove certain other restrictions. Certain additional duality relations are obtained. Shortly before writing this abstract, the writer became acquainted with the contents of a paper by E. R. van Kampen (Amsterdam Proceedings, vol. 31 (1928), pp. 899–905), which contains results of which Morse's second set of relations are a corollary, proved (in outline) in addition under broader hypotheses, and with additional results regarding intersection numbers. However, the proofs of the present paper are of simpler character, and in addition the author obtains certain duality relations not obtained by van Kampen. (Received March 4, 1932.)

82. Dr. Hassler Whitney (National Research Fellow): *Regular families of curves.* I.

A function of closed sets $\mu(S)$ is defined, such that if every point of S' is within ϵ of S, then $\mu(S') \leqq \mu(S)+2\epsilon$, and if $S \subset S'$, $S \neq S'$, then $\mu(S') > \mu(S)$. A family of curves is regular if, roughly, neighboring arcs of curves have a small écart distance. By using the function μ, a function $p' = g(p, \alpha)$ is defined, p' lying on the curve through p, which is continuous in both variables. The function also enables one to draw a cross section through any point of the family. Examples show the usefulness of the function in topology. (Received March 3, 1932.)

83. Dr. Hassler Whitney (National Research Fellow): *Regular families of curves.* II.

Let a regular family of curves form an open subset R' of a compact metric space R; the rest of R shall consist of "invariant points." It is shown how a sequence of "tubes" with certain properties fills out R'. A function $p' = f(p, t)$ is found with the following properties: (1) $f(p, t)$, $-\infty < t < +\infty$, forms exactly the curve through p, or is p, if p is an invariant point; (2) $f(p, t)$ is continuous

in both variables everywhere; (3) $f[f(p, t_1), t_2] = f(p, t_1+t_2)$; also $f(p, 0) = p$. (Received March 3, 1932.)

84. Professor W. A. Wilson: *A relation between metric and euclidean spaces.*

This paper deals principally with convex complete metric spaces which have the property that any four points are congruent with four points in euclidean space. Along with other results it is shown that if such a space is also separable and externally convex, it is congruent with some euclidean space or with Hilbert space. (Received March 2, 1932.)

85. Professor W. A. Wilson: *On angles in metric spaces.*

In this note conditions are given which are sufficient for the definition of a system of angles in certain metric spaces analogous to angles in euclidean spaces. Tangents to simple arcs are defined and an intrinsic condition is found, which is necessary and sufficient for the existence of a tangent in the spaces discussed. (Received March 1, 1932.)

86. Mr. Abraham Sinkov: *Families of groups generated by two operators of the same order.*

Except in a few special cases, a group generated by two operators is not completely determined by their orders and the order of some additional combination of them. The set of groups which is obtained as a solution to such conditions quite often forms what may be considered a family in the sense that the groups are all of the same type and have their orders expressible as an algebraic function of one or more parameters. The present paper obtains the defining relations and exhibits the generating operations of two simply isomorphic families of solvable groups whose orders involve three parameters. The order is given by the expression nm^{n-k} where k is an arbitrary factor of n; it includes as special cases a few of the families which have previously appeared in the literature. (Received February 29, 1932.)

87. Mr. Marshall Hall: *Significance of quadratic residues in factorization.*

This paper deals with the so called eventual quadratic residues of unknown composition (Kraitchik's notation). The laws governing their combinations are considered, and the correspondence between a number and its set of eventual residues is shown. A theoretical test for primality is given, and also a modified form for practical applications. An example is given in illustration. (Received March 5, 1932.)

88. Professor Oystein Ore: *Non-commutative polynomials.*

This paper contains a theory of polynomials in a non-commutative field K. The coefficients are usually not commutative with the variable x, but the condition is imposed that the degree of a product is equal to the sum of the degrees of the factors. By means of the notion of transformation, the polynomials are

classified in types, and three normal representations of polynomials are obtained. One of them is the decomposition in prime factors, which is usually not unique, and it may contain an infinite number of different factors. Among the applications of this theory are the formal theory of linear difference and differential equations, and the theory of equations in linear algebras. (Received March 3, 1932.)

89. Dr. J. L. Dorroh (National Research Fellow): *On the factorization of divisors of zero in finite commutative rings.*

This paper treats only finite rings which contain an identity under multiplication. Two divisors of zero are called associates of each other if each divides the other. A *prime* divisor of zero is one whose only factors are its associates and regular elements. It is known that a ring of the type considered is the direct sum of irreducible subrings and that the question of uniqueness in the decomposition of its divisors of zero into prime factors reduces to the same question for its irreducible subrings. The order of an irreducible finite ring is a power of a prime integer. It is shown that if the order of an irreducible finite ring R is p^k, then R contains just $p^{(r-1)k/r}$ divisors of zero, where r is a factor of k. Decomposition in R is unique if, and only if, R contains a nilpotent element of index r. Decomposition is unique in any finite ring whose order is not divisible by the cube of a prime integer. Some other types of finite rings are classified according to the uniqueness or ambiguity of the decomposition of their divisors of zero into prime factors. (Received March 4, 1932.)

90. Dr. Leonard Carlitz: *On factorable polynomials in several indeterminates over a Galois field.*

By a factorable polynomial is meant one that splits into linear factors in *some* Galois field. In this paper, a study is made of the distribution of factorable polynomials, particularly of irreducible polynomials; the expression obtained for the number of primary irreducible polynomials is a simple generalization of Gauss's formula for the case of a single indeterminate. Definitions are then framed for certain "arithmetic" functions of the polynomials; exact expressions are obtained for the sum of each of these functions taken over all polynomials of fixed degree. (Received March 2, 1932.)

91. Dr. Leonard Carlitz: *On an arithmetical functional equation.*

This paper is concerned with the functional equation $\sum\psi(d, x)\psi(\delta, y) = \psi(\nu, xy)$, the summation being taken over all d, δ with L.C.M. equal to ν. The equation is suggested in another paper of the author's (*On factorable polynomials in several indeterminates over a Galois field*) for the case $\psi(\nu, x) = \sum\mu(\nu/\delta)x^\delta$, summed over all δ dividing ν. The general equation is easily shown to be equivalent to a simple equation involving an arithmetic function of a single variable, whence the particular case above is very easily proved. It should be remarked that the second argument in $\psi(\nu, x)$ need not be integral. (Received March 2, 1932.)

92. Dr. Leonard Carlitz: *On a problem in additive arithmetic.* Second paper.

In a paper in the Quarterly Journal of Mathematics (Oxford Series, vol. 2 (1931), p. 97) the author, using the Hardy-Littlewood method of Farey dissection, obtained an asymptotic expression for $\sum \alpha(n_1) \cdots \alpha(n_\nu)$, summed over all partitions of n into $\nu(\nu \geqq 3)$ positive integers, $\alpha(n)$ being a factorable function satisfying certain conditions. In the present paper the more general sum, $\sum \alpha_1(n_1) \cdots \alpha_\nu(n_\nu)$, is considered, the $\alpha_k(n)$ being similar to $\alpha(n)$; by using an elementary method an asymptotic expression is found for *all* values of ν. (Received March 2, 1932.)

93. Professor Raymond Garver: *On the approximate solution of certain equations.*

A modification of Newton's method is presented which gives exact results for certain exponential equations instead of linear equations, and which gives very close approximations for many equations representing graphically similar functions. It is applied to the equation $x^x = c$, to the extraction of roots, and to other equations. (Received March 4, 1932).

94. Dr. B. F. Kimball: *The application of Bernoulli functions of negative order to differencing.*

In this paper the author calls attention to the formula (1) $\Delta^n f(x) = \sum_{m=0}^{\infty} (1/((2m)!)) B_{2m}^{-n}(-n/2) f^{(n+2m)}(x+n/2)$ as useful in studying the differences of a function (compare Nörlund, *Differenzenrechnung*, 1924, p. 142). It is shown that, for $n > m$, $B_{2m}^{-n}(-n/2)$ takes the form of a polynomial of degree m in n with coefficient of leading term in n equal to $(2m)!/((24)^m(m!))$. Several applications of (1) are given, among which is the determination of the asymptotic value of $\Delta^n \log x$ as n becomes infinite, x positive. This is shown to be $(\Gamma(x))/(n^x \log n)$. The results can be modified to apply to the case where difference intervals are equal to a real quantity w, rather than equal to 1. (Received March 5, 1932.)

95. Professor C. N. Moore: *On certain sufficient conditions that a set of constants should be Fourier constants.*

The relationship between functions whose squares are integrable (L) and their corresponding Fourier constants is extremely simple and elegant. A combination of the theorems of Parseval and Riesz-Fischer tells us that the convergence of the series $\sum (a_n^2 + d_n^2)$ is both a necessary and a sufficient condition that the function from which these constants have been obtained should be of integrable square. For the more general case of functions integrable (L) we have no such simple result. A necessary condition is that $\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n = 0$, but this is not sufficient. In the present paper it is shown that for the cosine series the conditions (1) $a_n \log n$ remains bounded, (2) $\sum a_n |\Delta^2 a_n|$ converges, are sufficient. These conditions are related to certain sufficient conditions derived by W. H. Young and Szidon, but they include cases of interest

not covered by the theorems of the writers mentioned. (Received February 19, 1932.)

96. Professor Edward Kasner: *Successive curvatures of dynamical trajectories.*

A familiar theorem states that the radius of curvature r of a trajectory varies inversely as the square of the speed v. The present author gives the extension of this result, for any field of force, to the successive radii $r_1, r_2, \cdots$ of the successive evolutes of the trajectory. A separate discussion has to be made for the case of initial motion in the direction of the force, since then $r = \infty$. The results are then stated in terms of curvature γ and its derivatives γ_s, γ_{ss}, γ_{sss}, $\cdots$. In the simplest case, γ varies as v^{-2}. A separate theory is also needed when a particle starts from rest, as considered in earlier papers by the author (see Princeton Colloquium Lectures, p. 9, for the simplest theorem, curvature of trajectory equals one third the curvature of line of force, and the application by Roever, this Bulletin, vol. 21 (1914–15), p. 444). (Received March 5, 1932.)

97. Professor J. M. Thomas: *Regular differential systems of the first order.*

The theorem that every passive regular differential system of the first order is completely integrable follows readily from a result of Riquier's (*Les Systèmes d'Equations aux Dérivées Partielles*, p. 350). The present paper, however, shows that it can be obtained from Cauchy's existence theorem by direct generalization of a method due to J. Koenig (Mathematische Annalen, vol. 23), without any new convergence demonstration being necessary. The theorem is moreover connected with Cartan's existence theorem for non-singular solutions of Pfaffian systems. The relation between Cartan's and Riquier's work is thus established. Cartan's result is also generalized to systems whose left members are symbolic forms of higher degree than the first. (Received March 5, 1932.)

98. Professor A. K. Mitchell: *On a matrix differential operator.*

H. W. Turnbull (*On differentiating a matrix*, Proceedings of the Edinburgh Mathematical Society, (2), vol. 1, Part 2 (1928)) has defined a matrix differential operator and given several theorems concerning its properties. In this note simplified direct proofs of four of these theorems are given, the simplification resulting from the use of tensor notation and the properties of the generalized Kronecker delta. Finally, the effect of the operator upon the coefficients of the characteristic equation of a matrix is considered. (Received February 12, 1932.)

99. Professor B. F. Kimball: *Three theorems applicable to vibration theory.*

In discussions of the theory of free vibrations it is sometimes stated without proof that "the fact that the period of vibration is independent of the amplitude

shows that the restoring force is linear." The present paper includes a proof of this statement and a brief consideration of the case of a non-linear restoring force of a fairly general nature. The origin is taken as the position of equilibrium; the force considered is a continuous function, positive with positive derivative for x positive. The force is zero at the origin with positive unilateral derivative there. It is taken symmetrical with respect to the origin. Under these conditions on the force, the statement quoted above is proved. We also establish the double inequality $4\cdot 2^{1/2}[a/f(a)]^{1/2}<T<4\cdot 2^{1/2}[a^2/F(a)]^{1/2}$, where T denotes the period of vibration, a the amplitude, $f(x)$ the force, and $F(a)=\int_0^a f(x)dx$. Adding to the above hypothesis the existence of unilateral second derivatives at the origin, we prove that the limiting value of the period as the amplitude of vibration approaches zero is $2\pi[f'(0)]^{-1/2}$. (Received March 1, 1932.)

100. Dr. S. S. Wilks (National Research Fellow): *Certain generalizations in the analysis of variance.*

By means of the distribution of the ratio of two independently distributed variances, R. A. Fisher has found, among other things, the distributions of the correlation ratio and of the multiple correlation coefficient in samples from a normal population of one variable in which these quantities are zero. Certain limiting forms of this distribution yield the χ^2 and Student's distributions. In the present paper this theory is extended to samples from a multivariate normal population, and the moments and distributions are found of the generalized forms of the variance, correlation ratio, and ratio of variances. Other results reached by the methods used in this paper include the moments and distributions of determinants and ratios of determinants of correlation coefficients, moments of the extension of the Pearson and Neyman λ criterion appropriate to testing the hypothesis that k samples are from the same multivariate normal population, and a new proof of Hotelling's generalization of Student's distribution. Moments of these statistics were found from Wishart's generalized product moment distribution (Biometrika, vol. 10A (1928)), and the expressions for their distributions were derived from the solutions of two integral equations. (Received February 24, 1932.)

101. Professor H. V. Craig: *Applications of a covariant differentiation process.*

This paper deals with certain invariants which are formally quite similar to the gradient, divergence, and curl. These invariants are defined by means of the process given in the paper entitled *On a covariant differentiation process*, this Bulletin, vol. 37 (1931), p. 731. (Received March 5, 1932.)

102. Dr. J. L. Doob: *The boundary values of analytic functions.*

If $f(z)$ is a function analytic and bounded for $|z|<1$, $\lim_{r\to\infty} f(re^{it})$ exists for almost all t on the interval $(0, 2\pi)$, defining the Fatou boundary function $F(z)=F(e^{it})$ almost everywhere on $|z|=1$. Let (1) $f_1(z), f_2(z), \cdots$ be a uniformly bounded sequence of functions analytic for $|z|<1$, with Fatou boundary functions (2) $F_1(z), F_2(z), \cdots$ respectively. Necessary and sufficient condi-

tions on the sequence (2) are found that the sequence (1) converge uniformly in every closed subregion of $|z|<1$. Let $f(z)$, $F(z)$ be as above, and suppose in addition that $f(z)$ is smooth. Then if P is a point on $|z|=1$, necessary and sufficient conditions on $F(z)$ in a neighborhood of P are found (a) that $F(z)$ be defined at P, (b) that $f(z)$ have a given complex number as one of its limit values at P when z approaches P from within $|z|<1$, (c) that this limit value be a limit value when z approaches P on a sequence of points lying in an angle whose sides are chords of $|z|<1$ meeting at P. (Received February 6, 1932.)

103. Dr. J. J. Gergen: *Convergence criteria for double Fourier series.*

In this paper generalizations to two variables are given of certain familiar criteria for the convergence of a simple Fourier series: they are, chiefly, the analogues of the various forms of the Lebesgue test and the Hardy-Littlewood test. The tests for two variables are strictly analogous in so far as the characteristic conditions are concerned, but the continuity conditions are more restrictive than in the original tests. The relations between the generalizations to two variables of the six commonly recognized criteria for a single variable are discussed. It is shown, in particular, that the Lebesgue test includes Tonelli's test for double series, which deals with functions of bounded variation. (Received March 4, 1932.)

104. Professor George Rutledge: *The inverse matrix for de la Vallée-Poussin summation.*

The equations $s_n'=\mu_{n0}u_0+\mu_{n1}u_1+\cdots+\mu_{nn}u_n$, defining the de la Vallée-Poussin sequence for the series $u_0+u_1+u_2+\cdots$, are solved for $u_0, u_1, u_2, \cdots$ by application of a relation between de la Vallée-Poussin summation and the Stirling interpolation series (Journal of Mathematics and Physics of the Massachusetts Institute of Technology, vol. 9 (1930), p. 261). (Received February 23, 1932.)

105. Professor George Rutledge: *A reliable method of obtaining the derivative function from smoothed data of observation.*

The essential tool in the process discussed in this paper is the interpolation polynomial (Lagrange) with coefficients in explicit form (Transactions of this Society, vol. 26 (1924), p. 113, and vol. 31 (1930), p. 807). Each value of the derivative function is obtained by retaining the figures common to three determinations the errors in which are normally not all of the same sign. Precise limits of error are established. The process has been used with marked success for obtaining the derivative function dp/dt for the function defined by the vapor pressure values of steam (F. G. Keyes and L. B. Smith, *Some final values for the properties of saturated and superheated water*, Mechanical Engineering, vol. 53 (1931), p. 132). (Received February 23, 1932.)

106. Dr. S. S. Wilks (National Research Fellow): *Estimates, moments, and distributions of certain statistics in fragmentary samples.*

In sampling from a multivariate population, it frequently happens that all of the items of a sample are not observed or classified with respect to all of the variates. In such cases questions arise concerning methods of estimating the population parameters from all of the observed data. In this paper, samples of N items are considered from a normal population of two variates x and y, in which s of the items are observed with respect to both x and y, m with respect to x only, and n with respect to y only. For given estimates of the means, approximations are obtained for the maximum likelihood estimates of the correlation coefficient and standard deviations of x and y in the population. The limiting forms of the variances and covariances of these estimates for large samples are found. Another system of estimates studied consists of those derived by maximum likelihood for each of the means, variances, and covariances, independently of the others. The characteristic function and an expression for the simultaneous sampling distribution were found for these estimates. (Received February 24, 1932.)

107. Dr. J. L. Doob: *Sequences of meromorphic functions.*

If $f(z)$ is a function meromorphic for $|z|<1$, we define its "cluster boundary function" $\mathfrak{F}(z)$ at each point of $|z|=1$ as the set of all limit values of $f(z)$ when z approaches that point from within $|z|<1$. Let (1) $f_1(z), f_2(z), \cdots$ be a sequence of functions meromorphic for $|z|<1$, with cluster boundary functions $\mathfrak{F}_1(z)$, $\mathfrak{F}_2(z), \cdots$ respectively. Let $\{A_n\}$ be a set of open or closed arcs on $|z|=1$. A point α will belong to the set s if there is a subsequence $\{f_{a_n}(z)\}$ and a sequence of points $\{z_{a_n}\}$ in $|z|<1$ such that $f_{a_n}(z_{a_n})\to\alpha$ and such that under the transformation $z'=(z-z_{a_n})/(\bar{z}_{a_n}z-1)$ the arc A_{a_n} is transformed into an arc A'_{a_n} on $|z|=1$ such that $\lim_{n\to\infty} mA'_{a_n}=2\pi$. A point α will belong to the set S if there is a subsequence $\{\mathfrak{F}_{a_n}(z)\}$ and a sequence of points $\{z_{a_n}\}$, z_{a_n} on the arc A_{a_n}, such that $\mathfrak{F}_{a_n}(z_{a_n})\to\alpha$, for some determination of $\mathfrak{F}_{a_n}(z_{a_n})$ for each value of n. If α is a point of s, but not a point of S, and if D is the domain containing α whose frontier points all belong to S, it follows that $D \subset s$ and that every point of D, except possibly two, is assumed by an infinite number of the functions (1). If there are two exceptions in D, every point except these two in the extended plane is assumed by an infinite number of the functions (1). (Received February 6, 1932.)

108. Mr. W. V. D. Hodge: *The Dirichlet problem for harmonic functionals.*

In this paper it is shown that if, in a euclidean n-cell with a regular boundary, we consider functionals of cycles of q dimensions which satisfy Volterra's condition for harmonic functionals, we can extend the methods of potential theory to show that such functionals are uniquely determined by assigned values on the q-cycles of the boundary, when these values are suitably restricted in respect to continuity. It is also shown that it is possible to define harmonic

functionals of the $(p-1)$-dimensional bounding cycles on a real closed analytic variety of $2p$ dimensions, which is subject to certain restrictions. It is proved that there are exactly R_p linearly independent functionals of the cycles which are harmonic everywhere on the variety, where R_p is the pth Betti number of the variety. (Received March 4, 1932.)

109. Mr. W. S. Lawton: *On the zeros of certain polynomials related to Laguerre and Jacobi polynomials.*

The object of this paper is to study the distribution of the zeros of the polynomials defined as follows: $J_n(x, \alpha, \beta) = x^{1-\alpha}(1-x)^{1-\beta}(d^n/dx^n)\left[x^{n+\alpha-1} \cdot (1-x)^{n+\beta-1}\right]$, $L_n(x, \alpha) = x^{1-\alpha}e^x(d^n/dx^n)\left[x^{n+\alpha-1}e^{-x}\right]$ for $\alpha, \beta \leqq 0$. (If $\alpha, \beta > 0$, these polynomials are known as Jacobi and Laguerre polynomials respectively, with all the zeros in the respective intervals $(0, 1)$, $(0, \infty)$.) As an illustration of the results obtained, we cite the following: if p and q are positive integers such that $0 < \alpha + p, \beta + q < 1$, then $J_n(x, \alpha, \beta)$ has *exactly* $n-p-q$ zeros inside the interval $(0, 1)$, and a similar result holds for $L_n(x, \alpha)$. (Received February 25, 1932.)

110. Mr. S. B. Myers: *Sufficient conditions in the problem of the calculus of variations in n-space in parametric form and under general end conditions.*

Sufficient conditions for a minimum in this general problem in parametric form are given here. The results are in terms of the characteristic roots of a linear boundary problem, and are in close relation to the conditions recently given by Morse in the corresponding problem in non-parametric form. An important feature of the results is that the usual "non-tangency" hypothesis is not made. For example, if these results were applied to the problem of minimizing an integral along curves joining a point to a manifold, we would obtain sufficient conditions for a minimum even in the case that the minimizing curve is tangent to the manifold. The essential idea in the methods used in the paper is the treatment of the parametric problem as the limiting case of a series of regular non-parametric problems by means of a suitable modification of the integrand. (Received February 27, 1932.)

111. Mr. Jacob Sherman: *On the numerators of the convergents of the Stieltjes continued fraction.*

This paper deals with the numerators of the convergents of the continued fractions "associated" with and "corresponding" to the Stieltjes integral $\int_a^b (1/(x-y))d\psi(y)$, $\psi(y)$ bounded and non-decreasing in the interval (a, b). Its main result states that *the numerators of both the even and odd convergents of the corresponding continued fraction*, the former being equal to the numerators of the associated continued fraction, *constitute systems of orthogonal polynomials.* For the associated continued fraction, the denominators of whose convergents are the classical orthogonal polynomials (of Jacobi, Laguerre, and Hermite), there is derived a linear differential equation, of the second order, for the numerators closely related to that for the denominators, including the result of Christoffel for the Legendre numerators as a very special case. Methods are

given for the expansion of the numerators in terms of the denominators with explicit formulas for the cases of Laguerre and Hermite. A relation is obtained between the characteristic function, $\psi(y)$, for the denominators and numerators, and also for other related functions dealt with in the theory of orthogonal Tchebycheff polynomials. The paper closes with some asymptotic evaluations for these new systems of polynomials. (Received February 18, 1932.)

112. Professor B. H. Brown: *Equiareal maps with parabolic meridians and parallels.*

In this paper the following problem is solved: to find all the area-preserving transformations in the plane which carry the rectilinear coordinate lines into a system of straight lines or parabolas with axes in a fixed direction. There are 21 distinct types of transformations, of which 8 are parabola-parabola systems, 11 line-parabola systems, and 2 line-line systems. Of these, 19 systems have the two families of different types, and 2 have them equivalent. There are thus 40 different possible maps. The general transformations are complicated, but can all be given explicitly in terms of algebraic functions. Only 3 of the 21 types appear to be old. Several of the new types seem well adapted for cartographic use. This is the first paper of a series dealing with general problems in area-preserving transformations. (Received February 20, 1932.)

113. Mr. George Comenetz: *The limit of the ratio of arc to chord for the curves drawn at a point on a surface.*

At an analytic point of a curve in complex space of three dimensions, the limit of the ratio of arc to chord, called the *rac*, is usually unity, but in special cases it can have any value whatever. Instead of the complete set of curves at a point in space, consider only the set of curves which lie on a surface analytic at the point. In the current issue of the Proceedings of the National Academy of Sciencies, Professor Kasner shows that the corresponding *rac set* will be one of five kinds, depending on the surface: (A) every complex value; (B) every value, with one real exception (the general case); (C) a discrete set of real values: 1.00, 0.94, 0.86, $\cdots$ (as in the ordinary plane); (D) unity and one other real value; (E) unity alone (as in a minimal plane). In a separate paper we shall give a geometrical classification of the curves at a point of a surface with respect to the value of the rac. The tangent plane at the point of the surface is of course either non-minimal or minimal; this distinction turns out to be a natural one for the problem. (Received February 10, 1932.)

114. Dr. S. S. Cairns: *The direction cosines of a p-plane, L_p, in a euclidean n-space, S_n.*

Let $(y_1, \cdots, y_n)$ denote rectangular cartesian coordinates in S_n. Consider a p-dimensional volume, V, on L_p. If the projection of V on the $(y_{i_1}, \cdots, y_{i_p})$-plane $(i_1 < i_2 < \cdots < i_p)$ is a p-dimensional volume, $V_{i_p \ldots i_1}$, let $\gamma_{i_1 \ldots i_p} = V_{i_p \ldots i_1}/V$. Otherwise let $\gamma_{i_1 \ldots i_p} = 0$. The $\binom{n}{p}$ ratios $\gamma_{i_1 \ldots i_p}$ will be called direction cosines of L_p. The sum of the squares of the γ's is unity. In all, there are $\binom{n}{p} - n(n-p)$ independent relations among the γ's, analogous to Vahlen's

identities (*Ueber die Relationen zwischen den Determinanten einer Matrix.* Journal für Mathematik, vol. 112 (1893)). For, if L_p is represented by $y_i = \Sigma_j^{p-1} a_{ij} u_j + y_i^0 (i = 1, \cdots, n)$, then $\gamma_{i_1 \cdots i_p} = |\delta_{i_1 \cdots i_p}/\Delta|$, where $\delta_{i_1 \cdots i_p}$ is the determinant of the rows $i_1, \cdots, i_p$ of the matrix $\|a_{ij}\|$, and Δ^2 is the sum of the squares of the δ's. The following theorem on matrices underlies the above results: Let $A_{st} = \Sigma_{i=t}^{n} a_{is} a_{it}$ $(s = 1, \cdots, p;\ t = 1, \cdots, p)$. Then $\Delta^2 = \Sigma' \pm (A_{1 \cdots}$ $\cdots, A_{p j_p})$, where Σ' is a summation over all permutations $(j_1, \cdots, j_p)$ of $(1, \cdots, p)$, the sign being plus for even and minus for odd permutations. If, in particular, $A_{st} = 1$ or 0, according as $s = t$ or $s \neq t$, then $\Delta^2 = 1$. (For $n = p$, the last sentence is a known theorem on determinants.) (Received March 5, 1932.)

115. Mr. J. M. Clarkson: *Some involutorial line transformations interpreted as points of V_2 of S_5.*

We consider four arbitrary pencils (A_i, α_i) of lines, in planes α_i, vertices A_i. A transversal line t determines a unique line of each, and these four lines have a second transversal t'. The involutorial transformation t, t' is interpreted in terms of points of a quadric hypersurface in S_5, and its properties are determined. Similarly, two pencils and a quadratic regulus define another involution; a third is obtained by taking two independent reguli. The images of ruled surfaces, congruences, and complexes are determined in each case. (Received February 25, 1932.)

116. Professor Louise D. Cummings: *Heptagonal systems of eight lines in a plane.*

The present investigation concerns the determination of the non-equivalent systems of eight real lines in a plane when no three of the lines are co-punctual. Two systems are equivalent if a one-to-one relation exists between the lines and the polygons of the two systems. This paper is limited to that subdivision of the problem where, in a set of eight lines, some seven form a convex heptagon. Each line is uniquely defined by its *mark*, which shows the arrangement of contiguous segments on the line. The eight marks are determined for each system and are tabulated together with the numbers of polygons of 3, 4, 5, 6, 7, 8 sides which occur in the system. The marks of the system are used to establish the non-equivalence of the eleven systems derived in this paper, and to determine the substitutions which transform the system into itself. (Received March 1, 1932.)

117. Professor Malcolm Foster: *On the inversion of a special cyclic system.*

This paper is concerned with the following cyclic system lying in the xz-planes of the moving trihedral of a surface S: $(x-R)^2+z^2=R^2$, $y=0$. If these circles are inverted relative to the vertex of the trihedral, we obtain a rectilinear congruence $x = k^2/(2R)$, $y=0$. The paper discusses the condition that the rectilinear congruence be also normal, and shows that two of the focal points of the cyclic system are collinear with one focal point of the rectilinear congruence. The inversion of the cyclic system $x^2+(z-R)^2=R^2$, $y=0$, is also considered. (Received March 3, 1932.)

118. Professor P. F. Smith: *On planar element strips.*

A strip of planar elements is a union of planar (surface) elements along a curve as defined by Sophus Lie. The differential geometry (classical) of element strips in space includes that of curves in spaces, and by it many theorems for surfaces are established. An element strip has three characteristic functions by which it is determined except as to position in space. Equations in series form for a strip are obtained, and also equations involving quadratures for certain special strips. Simultaneous differential invariants are derived for a strip and a surface upon which the strip lies. Two moving trihedrals (the tangent trihedral and the conjugate trihedral) are employed to give dual theorems. Spheres on three (four) consecutive points of strips are studied, and, dually, spheres on three (four) consecutive planes. The problem of two associated strips is discussed, and two examples (dual) when the strips have for corresponding elements a common center and radius of first (second) curvature are solved. (Received February 20, 1932.)

ERRATUM

Volume 37, page 818, abstract No. 343 (by L. E. Ward): substitute for the last sentence the following: "Conditions are given for the formal series for a function to converge uniformly to that function." The form in which this sentence appeared was due to clerical error.

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OFFICIAL COMMUNICATIONS

Meetings of the Society have been fixed at the following times and places:

NEW YORK CITY, March 25-26, 1932.

Abstracts must be in the hands of Associate Secretary Tomlinson Fort, 501 West 116th St., New York City, not later than March 5. All such abstracts will appear in the March issue of the BULLETIN. By invitation of the program committee, there will be a symposium on *Summability of series*, at which Professor Richard Courant will speak on *Functional methods and characteristic values in the calculus of probability*, Professor Einar Hille on *Summation of Fourier series*, Professor W. A. Hurwitz on *General theorems on linear transformation of sequences*, and others will participate.

CHICAGO, April 8-9, 1932.

Abstracts must be in the hands of Associate Secretary Mark H. Ingraham, University of Wisconsin, Madison, Wisconsin, not later than March 18. All those received on or before March 6 will appear in the March issue of the BULLETIN. By invitation of the program committee, Professor R. L. Wilder will deliver an address entitled *Point sets in three and higher dimensions and their investigation by means of unified analysis situs.*

On account of the fact that the Summer Meeting of the Society in 1932 will be held on the Pacific Coast, the meetings that were to have been held at Stanford University (April, 1932), at Seattle (June, 1932), and at Los Angeles (November, 1932) will be omitted.

LOS ANGELES, CALIFORNIA, SUMMER MEETING AND COLLOQUIUM, August 30-September 2, 1932.

Abstracts must be in the hands of Associate Secretary M. H. Ingraham, University of Wisconsin, Madison, Wis., not later than July 26. Abstracts received by July 7 will appear in the July issue, or an earlier issue, of this BULLETIN. Professor J. F. Ritt will deliver a series of colloquium lectures on *Systems of algebraic differential equations.*

NEW YORK CITY, October 29, 1932.

Abstracts are due by October 8, 1932.

R. G. D. RICHARDSON, *Secretary of the Society.*

Articles for insertion in the BULLETIN should be addressed to E. R. HEDRICK, Editor of the BULLETIN, University of California at Los Angeles. Reviews should be sent to W. R. LONGLEY, Yale University, New Haven, Conn. Notes should be sent to H. W. KUHN, Ohio State University, Columbus, Ohio.

Subscriptions to the BULLETIN, orders for back numbers, and inquiries in regard to non-delivery of current numbers should be addressed to the American Mathematical Society, 450-459 Ahnaip St., Menasha, Wis., or 501 West 116th St., New York.

The initiation fees and the annual dues of members of the Society (see this BULLETIN, p. 322, May, 1930; and the List of Officers and Members, October, 1930, p. 58), are payable to the Treasurer of the Society, Professor G. W. Mullins, 501 West 116th St., New York City.

WHOLE NUMBER 390

CONTENTS

PAGE

The Annual Meeting in New Orleans. By TOMLINSON FORT 145

The Scientific Work of C. L. E. Moore. By D. J. STRUIK 155

Enciclopedia delle Matematiche Elementari. By G. A. MILLER 157

Loria on History of Geometry. By DAVID EUGENE SMITH 159

Brunet on the Newtonian Movement. By DAVID EUGENE SMITH 161

Coolidge on Algebraic Curves. By VIRGIL SNYDER 163

Shorter Notices 166

Notes 174

Abstracts of Papers 177

New Publications 195

For official Communications and Notices, see the inside of the back cover.

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